The size of an atom

Suppose we have a hydrogen atom, and measure the position of the electron; we must not be able to predict exactly where the electron will be, or the momentum spread will then turn out to be infinite. Every time we look at the electron, it is somewhere, but it has an amplitude to be in different places so there is a probability of it being found in different places. These places cannot all be at the nucleus; we shall suppose there is a spread in position of order a. That is, the distance of the electron from the nucleus is usually about a. We shall determine a by minimizing the total energy of the atom.

The spread in momentum is roughly h/a because of the uncertainty relation, so that if we try to measure the momentum of the electron in some manner, such as by scattering x-rays off it and looking for the Doppler effect from a moving scatterer, we would expect not to get zero every time — the electron is not standing still — but the momenta must be of the order $p \approx$ (a   ). Then the kinetic energy is roughly $\frac{1}{2}mv^2$ = (b   ) $= h^2/2ma^2$. (In a sense, this is a kind of dimensional analysis to find out in what way the kinetic energy depends upon Planck's constant, upon m, and upon the size of the atom. We need not trust our answer to within factors like 2, π, etc. We have not even defined a very precisely.) Now the potential energy is minus e^2 over the distance from the center, say (c   ), where e^2 is the charge of an electron squared, divided by $4\pi\varepsilon_0$. Now the point is that the potential energy is reduced if a gets smaller, but the smaller a is, the higher the momentum required, because of the uncertainty principle, and therefore the higher the kinetic energy. The total energy is

E = (d   ).   (2.10)

We do not know what a is, but we know that the atom is going to arrange itself to make some kind of compromise so that the energy is as little as possible. In order to minimize E, we differentiate with respect to a, set the derivative equal to zero, and solve for a. The derivative of E is

dE/da = (e   ),   (2.11)

and setting $dE/da = 0$ gives for a the value

a_0 = (f   )

= (g   ) meter

= (h   ) angstrom.   (2.12)

This particular distance is called the *Bohr radius*, and we have thus learned that atomic dimensions are of the order of angstroms, which is right. This is pretty good — in fact, it is amazing, since until now we have had no basis for understanding the size of atoms! Atoms are completely impossible from the classical point of view, since the electrons would spiral into the nucleus.

Now if we put the value (2.12) for a_0 into (2.10) to find the energy, it comes out

E_0 = (h   ) = (i   )

= (j   ) .   (2.13)

What does a negative energy mean? It means that the electron has less energy when it is in the atom than when it is free. It means it is bound. It means it takes energy to kick the electron out; it takes energy of the order of 13.6 eV to ionize a hydrogen atom. We have no reason to think that it is not two or three times this — or half of this — or $(1/\pi)$ times this, because we have used such a sloppy argument. However, we have cheated, we have used all the constants in such a way that it happens to come out the right number! This number, 13.6 electron volts, is called a Rydberg of energy; it is the ionization energy of hydrogen.

R.P. Feynman, R.B. Leighton, and M.L. Sands: *The Feynman Lectures on Physics* (Addison-Wesley, 1965) Vol. III, Sec. 2-4 “The size of an atom”.