日本物理学会　第69回年次大会
28pBF-3 チュートリアル講演

ゼロギャップ半導体における スピン物性とワイル半金属相

野村健太郎　（東北大学金属材料研究所）

outline

1. What is Weyl semimetal
2. How can it be realized
3. Charge transport
4. Novel phenomena

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紅林大地(M1)

関根聡彦(D2)

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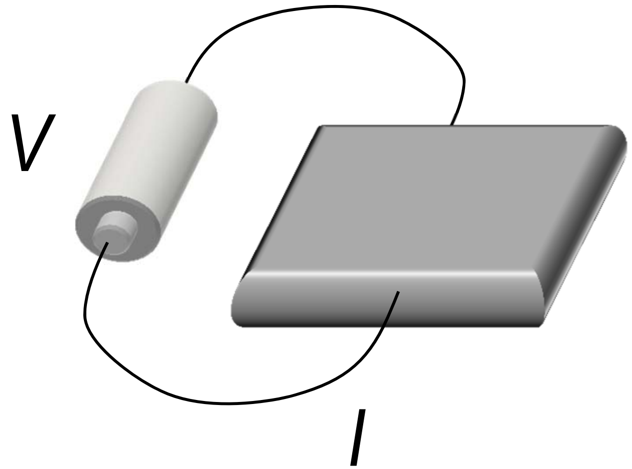
Insulator and Metal



$$g = \frac{I}{V}$$

conductance

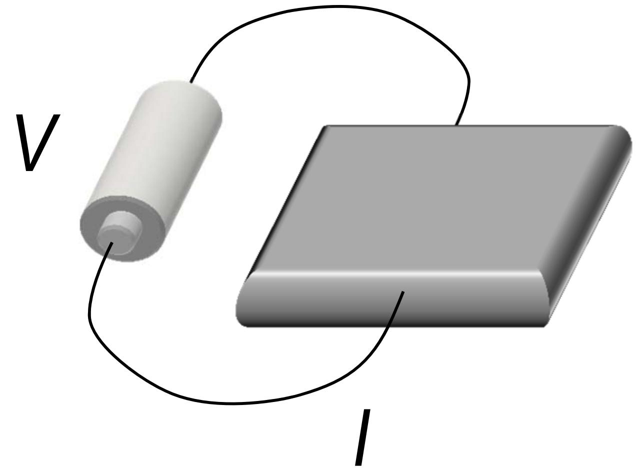
insulator

metal

small g

large g

Insulator and Metal



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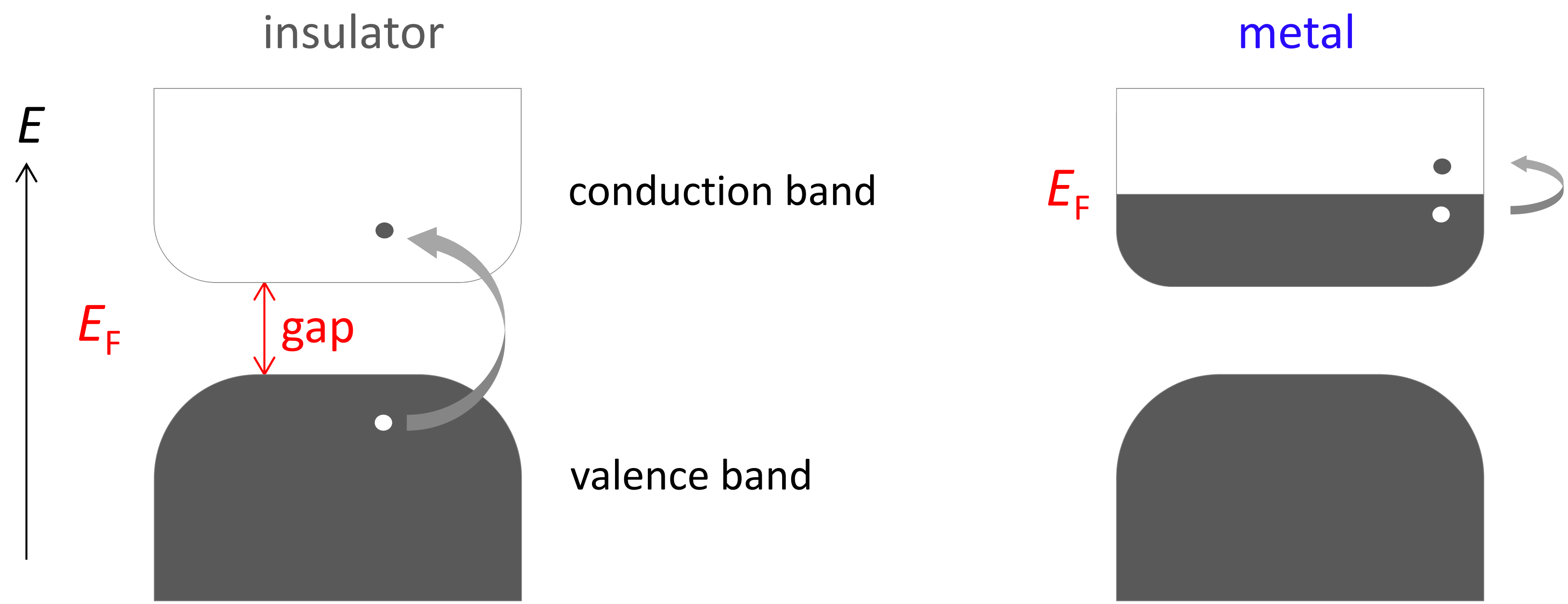
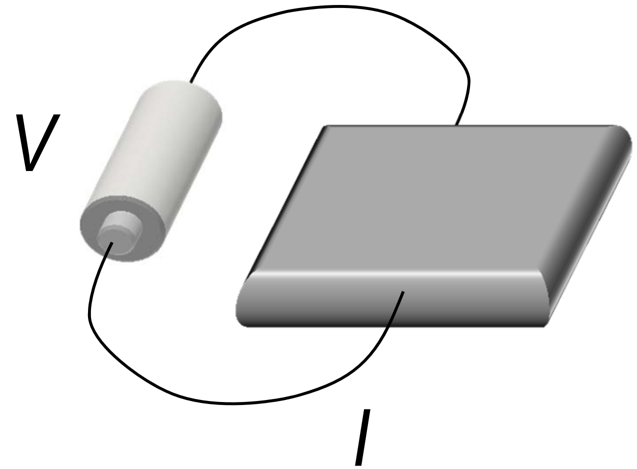
conductance

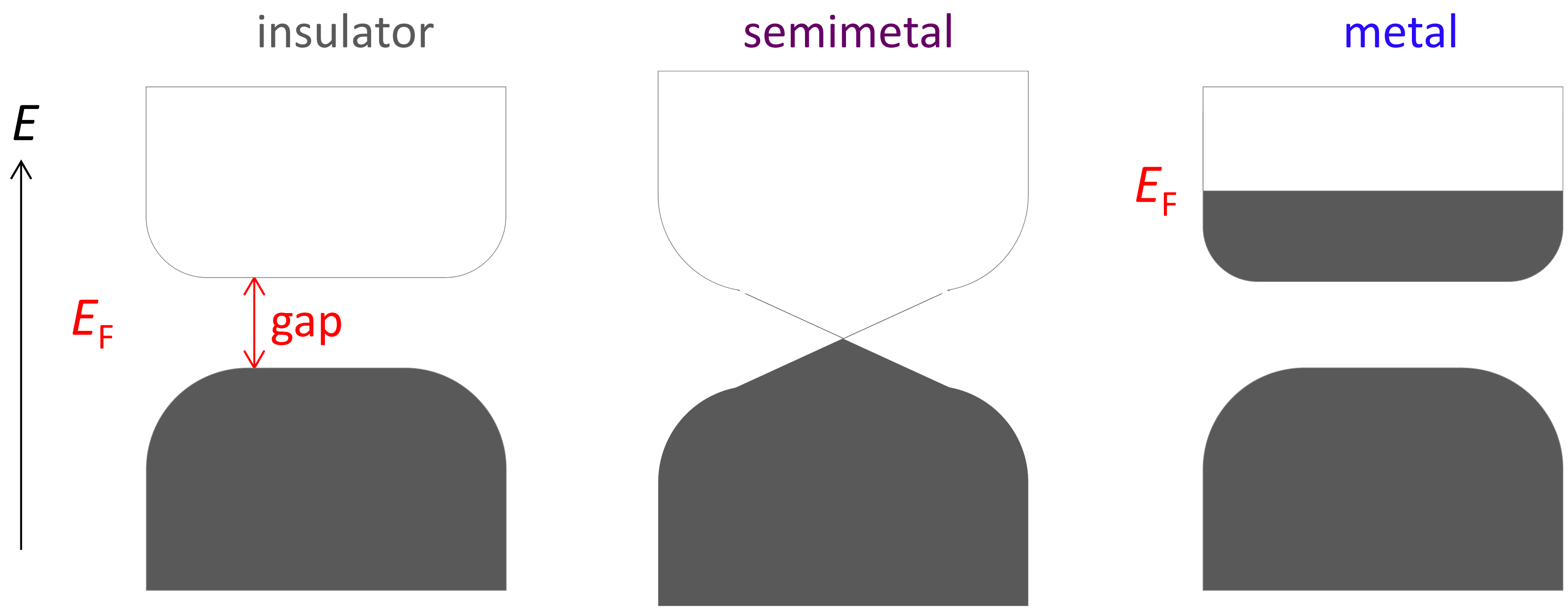


Insulator and Metal



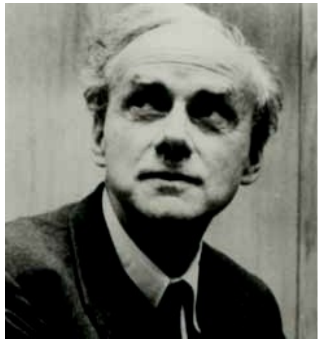
$$g = \frac{I}{V}$$

conductance



Relativistic quantum mechanics

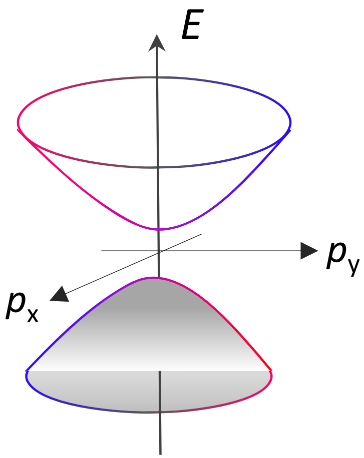
Dirac Fermions



$$H_{Dirac} = p_x\alpha_1 + p_y\alpha_2 + p_z\alpha_3 + m\alpha_4$$

α_i: 4x4 Dirac matrix

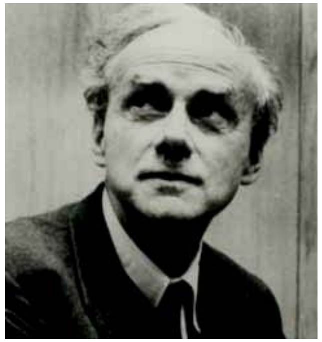
$$\{\alpha_i, \alpha_j\} = 2\delta_{ij}$$



$$E^2 = p_x^2 + p_y^2 + p_z^2 + m^2$$

Relativistic quantum mechanics

Dirac Fermions



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Weyl Fermions

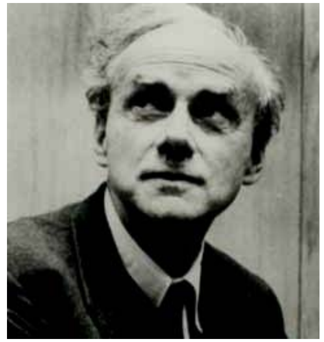
$$H_{Weyl} = p_x\sigma_1 + p_y\sigma_2 + p_z\sigma_3$$

σ_i: 2x2 Pauli matrix

-Massless fermions
-Parity broken

Relativistic quantum mechanics

Dirac Fermions



$$H_{Dirac} = p_x\alpha_1 + p_y\alpha_2 + p_z\alpha_3 + m\alpha_4$$

α_i: 4x4 Dirac matrix

$$H_{Weyl} = \begin{pmatrix} p_z & p_x - ip_y \\ p_x + ip_y & -p_z \end{pmatrix}$$

Weyl Fermions

$$H_{Weyl} = p_x\sigma_1 + p_y\sigma_2 + p_z\sigma_3$$

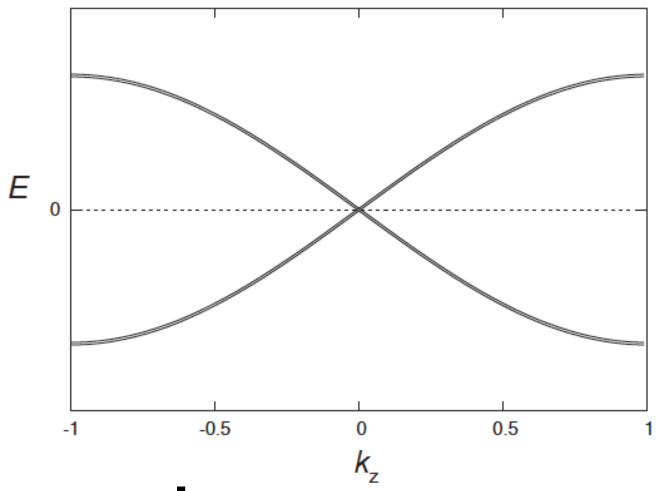
σ_i: 2x2 Pauli matrix

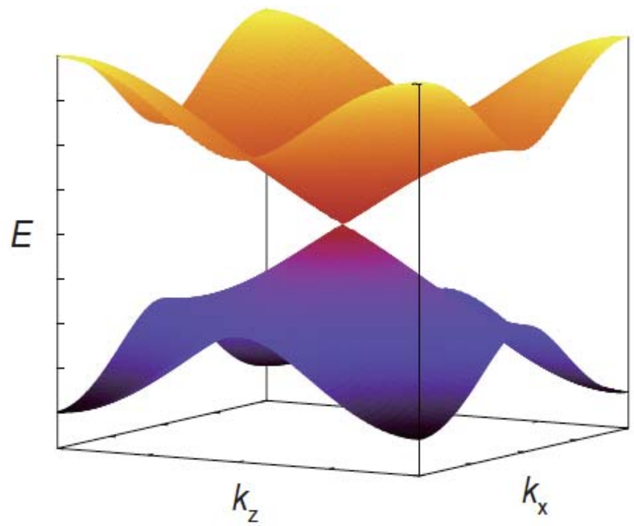
-Massless fermions
-Parity broken

Condensed matter systems

Dirac semimetals

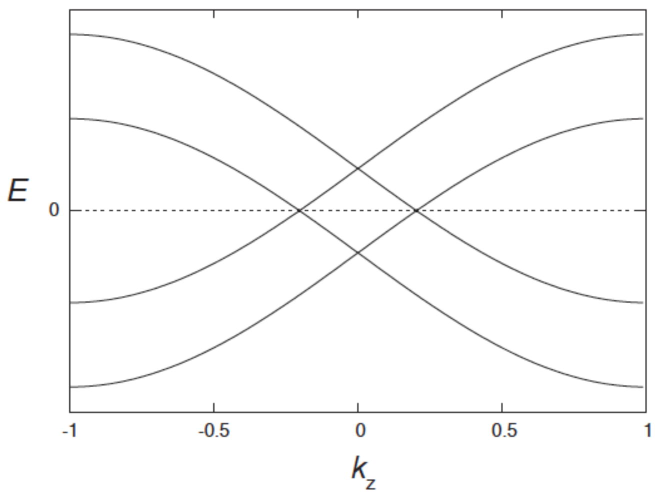
degenerate

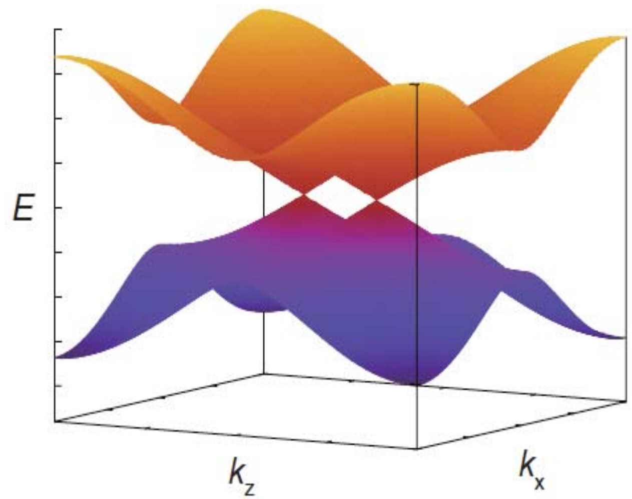




Weyl semimetals

non-degenerate

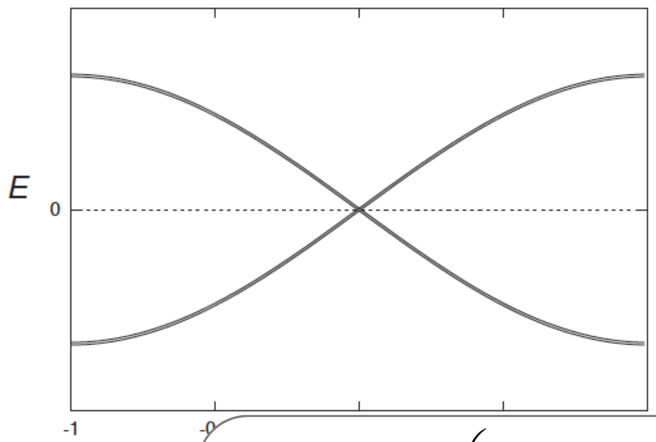


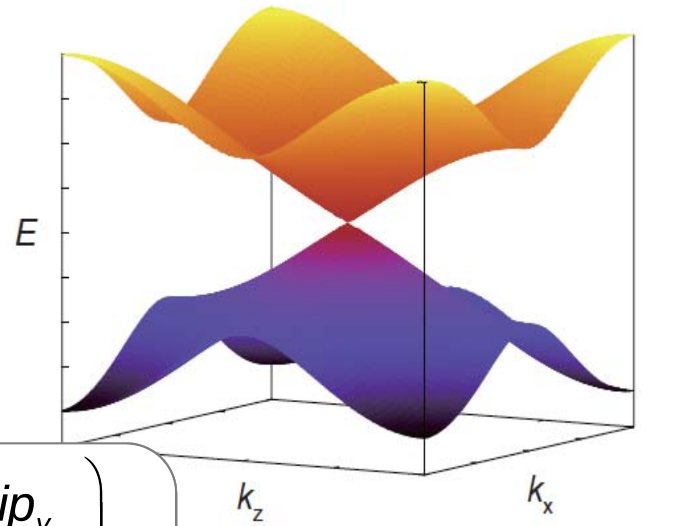


Condensed matter systems

Dirac semimetals

degenerate



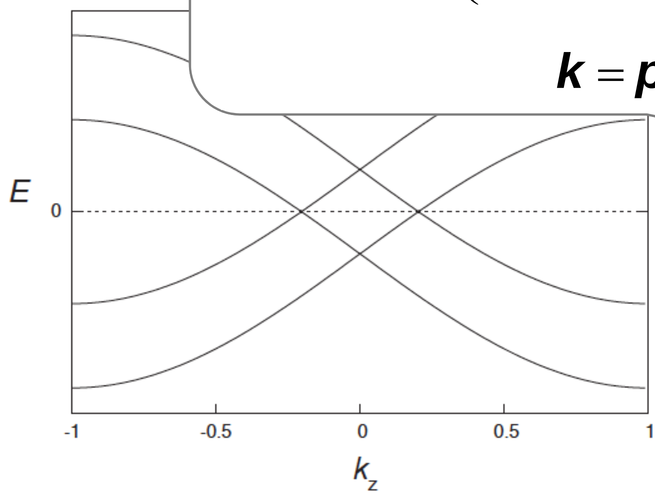


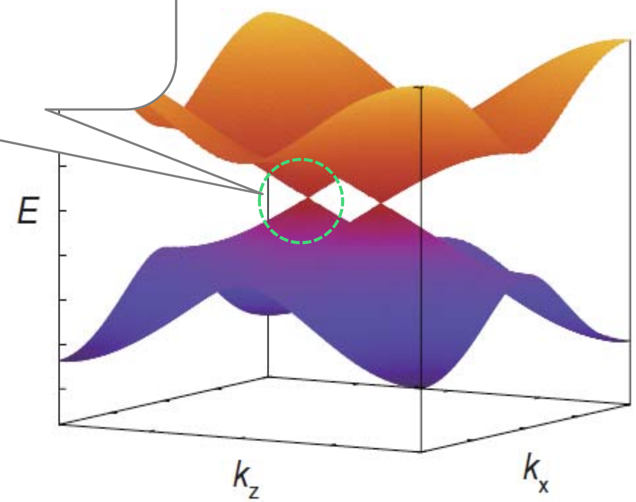
Weyl semimetals

$$H_{Weyl}^{(2x2)} = \pm \begin{pmatrix} p_z & p_x - ip_y \\ p_x + ip_y & -p_z \end{pmatrix}$$

$$\boldsymbol{k} = \boldsymbol{p} + \boldsymbol{K}_0$$

non-degenerate

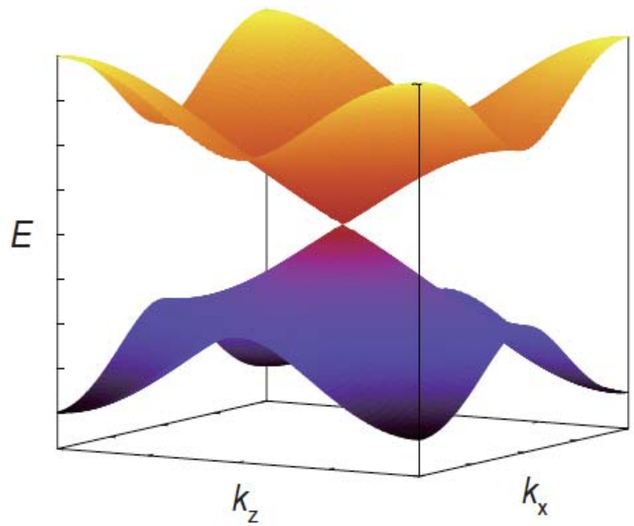




Condensed matter systems

Dirac semimetals

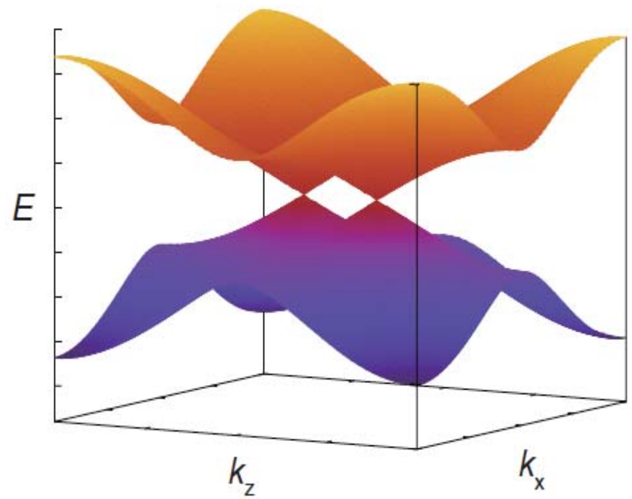
degenerate



2D (graphene)
  Wallace (1947), ...
3D (accidental)
  Herring (1937), ...
  (symmetry protected)
  Wang et al., Young et al.(2012),..

Weyl semimetals

non-degenerate



3D (I-broken)
  Murakami (2007)
3D (T-broken)
  Wan et al. (2011)
  Burkov&Balents (2012)

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Dirac semimetals

PHYSICAL REVIEW B **85**, 195320 (2012)

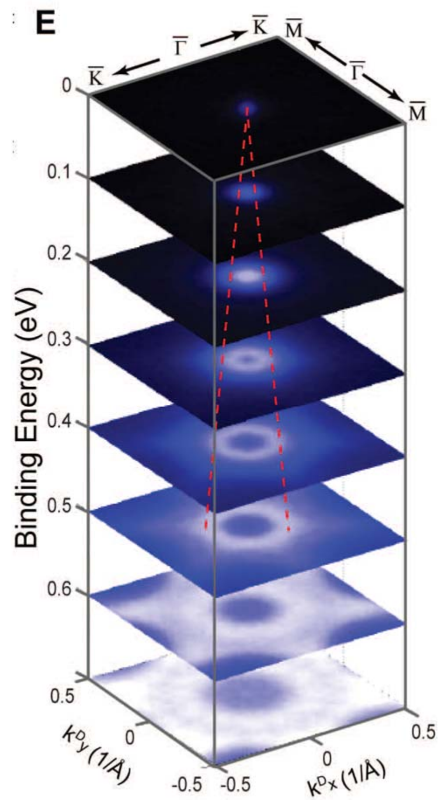
Dirac semimetal and topological phase transitions in A_3Bi (A = Na, K, Rb)

Zhijun Wang,[1] Yan Sun,[2] Xing-Qiu Chen,[2] Cesare Franchini,[2] Gang Xu,[1] Hongming Weng,[1,*] Xi Dai,[1] and Zhong Fang[1,†]
[1]*Beijing National Laboratory for Condensed Matter Physics and Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China*
[2]*Shenyang National Laboratory for Materials Science, Institute of Metal Research, Chinese Academy of Sciences, Shenyang 110016, China*
(Received 6 March 2012; revised manuscript received 7 May 2012; published 22 May 2012)

Discovery of a Three-Dimensional Topological Dirac Semimetal, Na_3Bi

Z. K. Liu,[1*] B. Zhou,[2,3*] Y. Zhang,[3] Z. J. Wang,[4] H. M. Weng,[4,5] D. Prabhakaran,[2] S.-K. Mo,[3] Z. X. Shen,[1] Z. Fang,[4,5] X. Dai,[4,5] Z. Hussain,[3] Y. L. Chen[2,6]†

Science 343, 864 (2014)

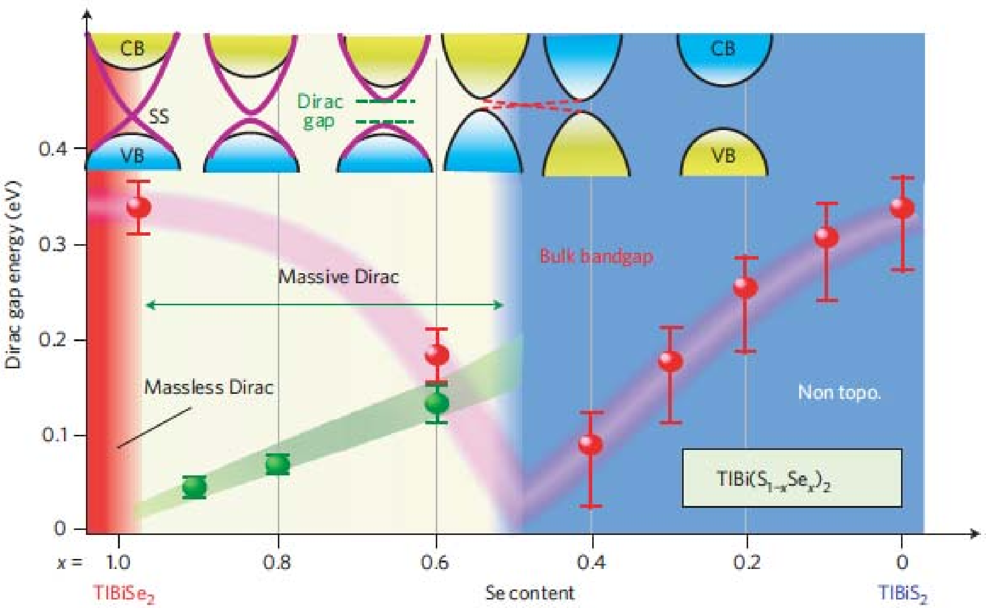


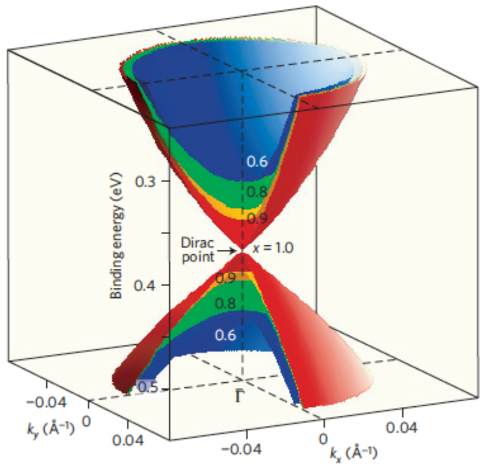
Dirac semimetals

Unexpected mass acquisition of Dirac fermions at the quantum phase transition of a topological insulator

T. Sato[1]*, Kouji Segawa[2], K. Kosaka[1], S. Souma[3], K. Nakayama[1], K. Eto[2], T. Minami[2], Yoichi Ando[2]* and T. Takahashi[1,3]

Nat. Phys. 7, 840 (2011)





Weyl semimetals

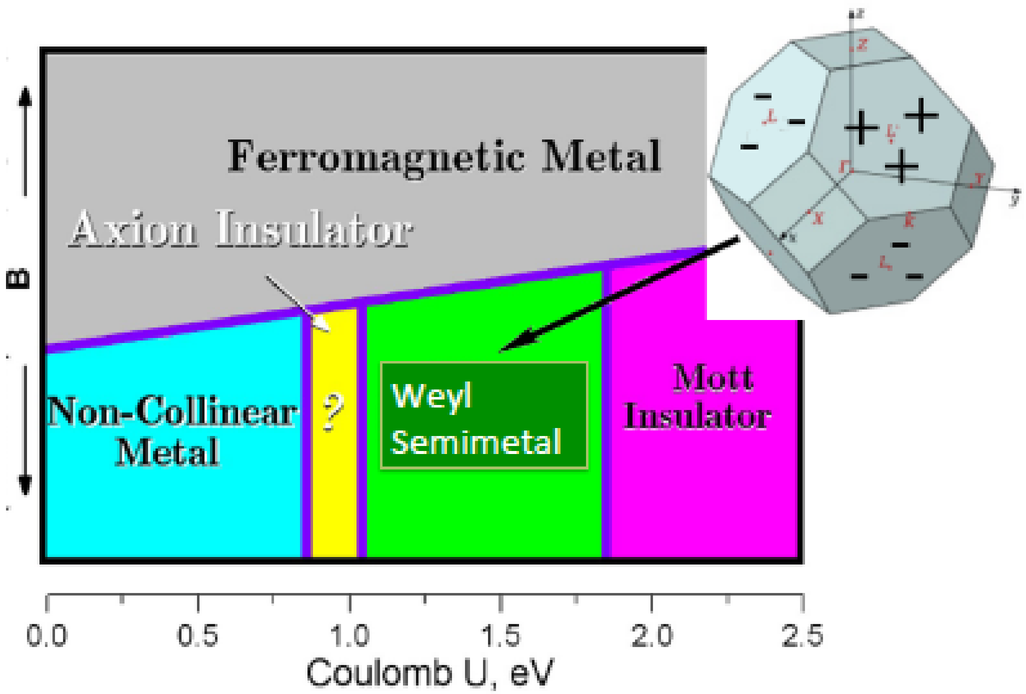
Selected for a Viewpoint in *Physics*

PHYSICAL REVIEW B **83**, 205101 (2011)

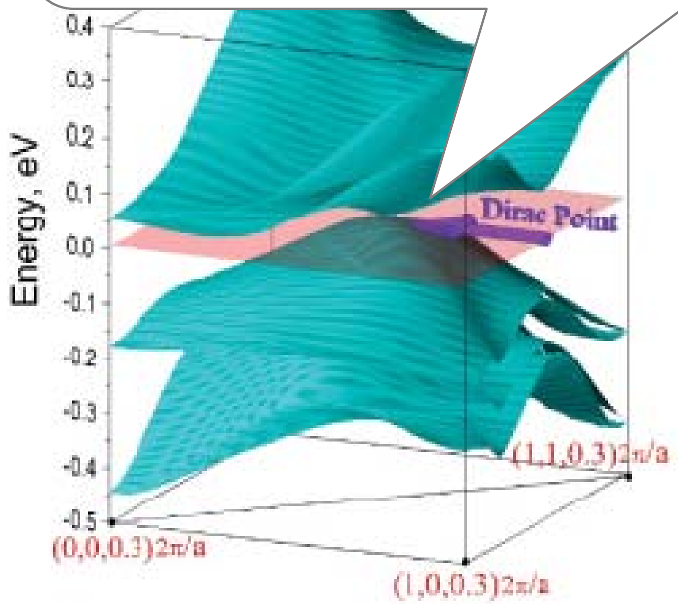
Topological semimetal and Fermi-arc surface states in the electronic structure of pyrochlore iridates

Xiangang Wan,[1] Ari M. Turner,[2] Ashvin Vishwanath,[2,3] and Sergey Y. Savrasov[1,4]

Pyrochlore iridates ($Y_2Ir_2O_7$) LDA+U



$$H_{Weyl}^{(2x2)} = \pm \begin{pmatrix} k_z & k_x - ik_y \\ k_x + ik_y & -k_z \end{pmatrix}$$



Weyl semimetals

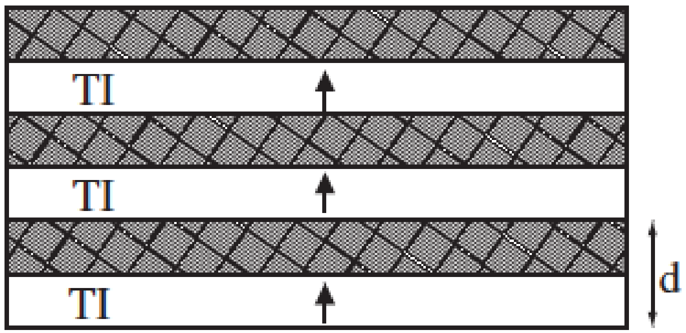
PRL **107**, 127205 (2011) PHYSICAL REVIEW LETTERS week ending 16 SEPTEMBER 2011

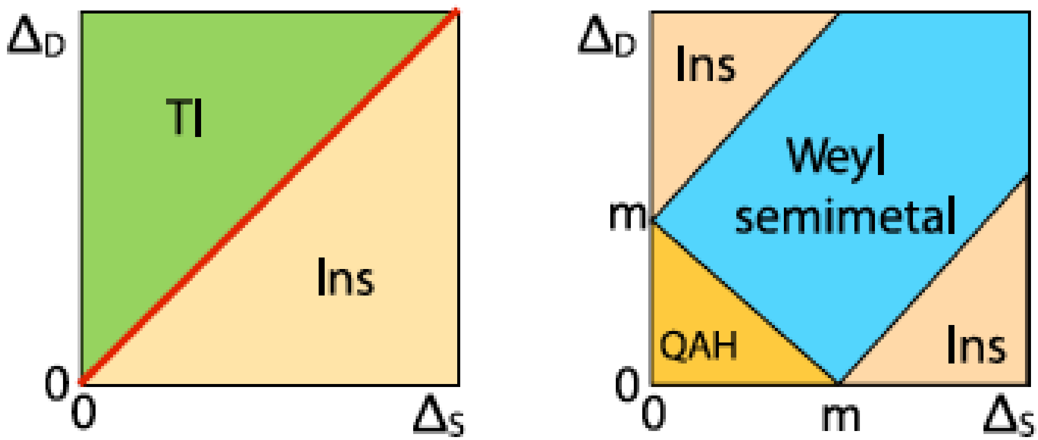
Weyl Semimetal in a Topological Insulator Multilayer

A. A. Burkov[1,2] and Leon Balents[2]
[1]*Department of Physics and Astronomy, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada*
[2]*Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California 93106, USA*
(Received 27 May 2011; published 16 September 2011)

$$\mathcal{H}(\mathbf{k}) = v_F \tau^z (\hat{z} \times \boldsymbol{\sigma}) \cdot \mathbf{k} + m\sigma^z + \hat{\Delta}(k_z),$$

$$\hat{\Delta} = \Delta_S \tau^x + \tfrac{1}{2}(\Delta_D \tau^+ e^{ik_z d} + \text{H.c.})$$





(a) m=0 (b) m≠0

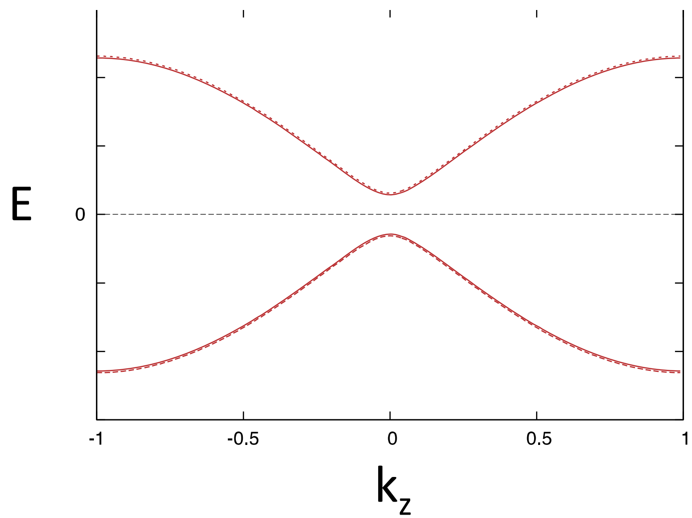
From Dirac to Weyl

Dirac hamiltonian

$$H = \sum_{i=1}^{3} p_i \alpha_i + m_0 \alpha_4$$

$$\alpha_i = \begin{bmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{bmatrix}, \qquad \alpha_4 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$$

degenerate



From Dirac to Weyl

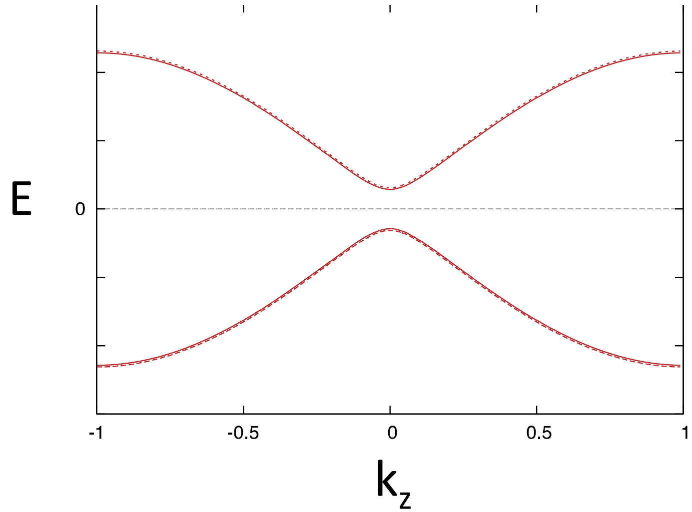
Dirac hamiltonian

$$H = \sum_{i=1}^{3} p_i \alpha_i + m_0 \alpha_4 + b\Sigma_z$$

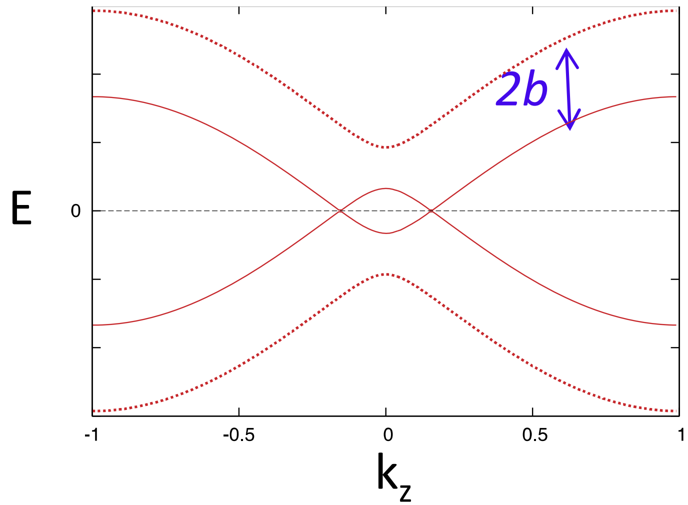
$$\alpha_i = \begin{bmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{bmatrix}, \qquad \alpha_4 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$$

$$\Sigma_i = \begin{bmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{bmatrix}$$ ← spin of electron

degenerate



non-degenerate

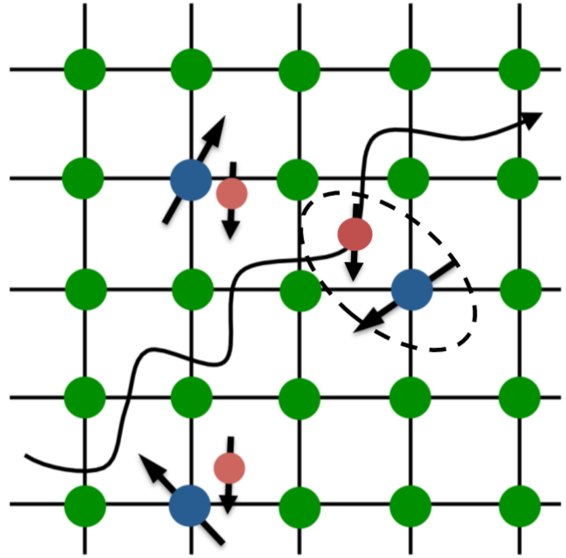


From Dirac to Weyl

Dirac hamiltonian

$$H_J = J\sum_I \boldsymbol{S}(\boldsymbol{r}_I)\cdot c_I^\dagger \boldsymbol{\Sigma} c_I$$

$$H = \sum_{i=1}^{3} p_i\alpha_i + m_0\alpha_4 + JM\Sigma_z$$

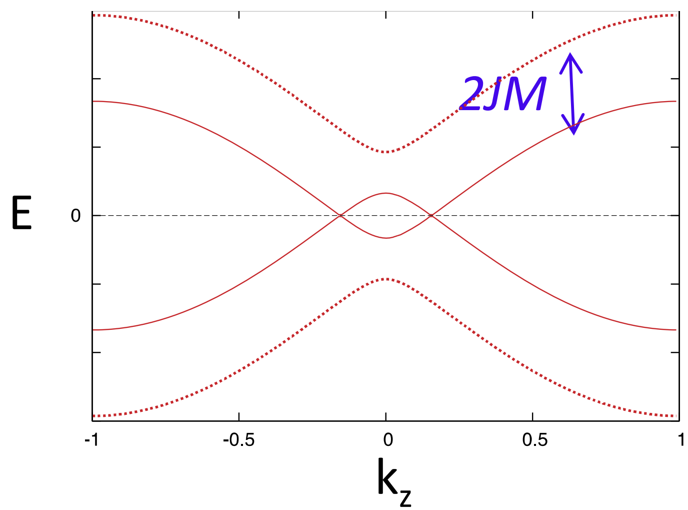


$$\alpha_i = \begin{bmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{bmatrix}, \qquad \alpha_4 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$$

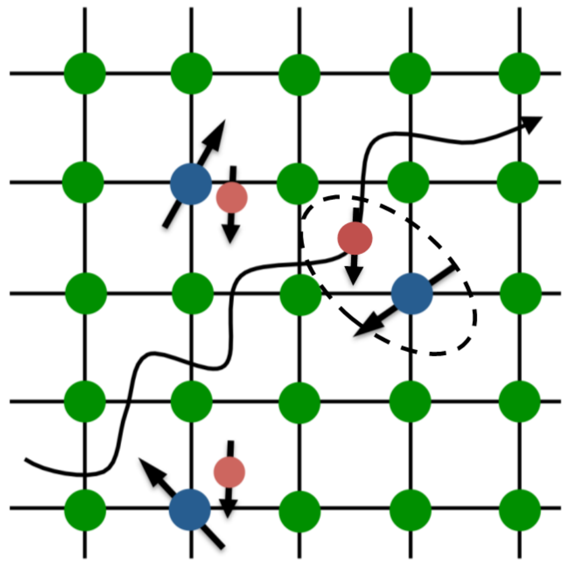
$$\Sigma_i = \begin{bmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{bmatrix}$$

← spin of electron

non-degenerate



Magnetic order?



Free energy

local spin → M^2 ; electron → m^2 ; interaction → JMm

$$F = \frac{1}{2\chi_L} M^2 + \frac{1}{2\chi_e} m^2 - JMm$$

$$= \frac{1}{2\chi_L}\left(1 - J^2 \chi_e \chi_L\right) M^2 + \frac{1}{2\chi_e}\left(m - \chi_e J M\right)^2$$

$$1 - J^2 \chi_L \chi_e < 0 \quad \Rightarrow \quad M \neq 0$$

$\chi_L \sim 1/T$ at low temperature

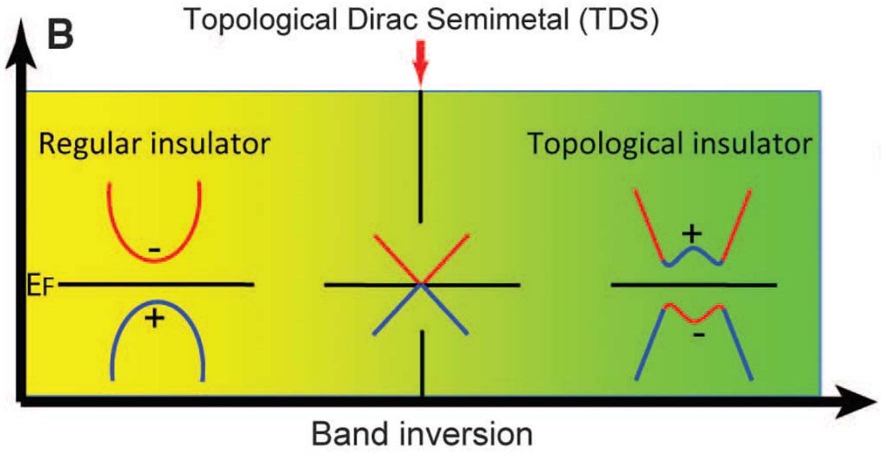
Magnetic order?

$$\chi_e = \sum 4\mu_0\mu_e\left[f(E_{n,k}) - f(E_{m,k})\right]\frac{\langle u_{n,k}|s_z|u_{m,k}\rangle\langle u_{m,k}|s_z|u_{n,k}\rangle}{E_{m,k}-E_{n,k}}$$

conduction band

valence band

Van-Vleck paramagnetism -> χ_e finite



$$1 - J^2\chi_L\chi_e < 0 \quad \Rightarrow \quad M \neq 0$$

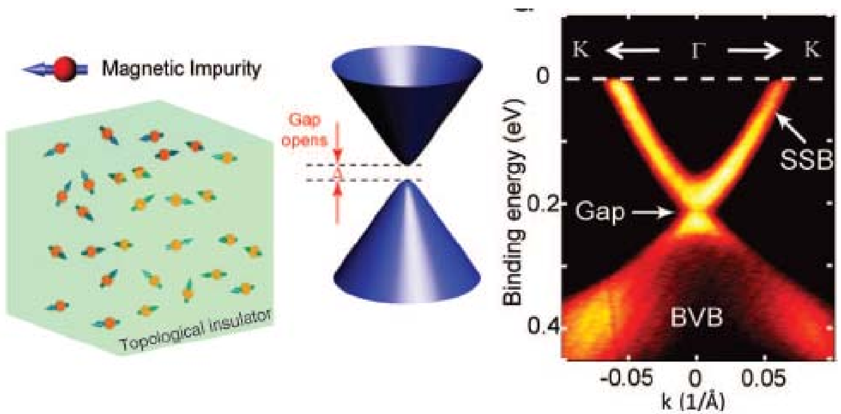
$\chi_L \sim 1/T$ at low temperature

Experiments of magnetic TIs

Science 329, 659 (2010)

Massive Dirac Fermion on the Surface of a Magnetically Doped Topological Insulator

Y. L. Chen,[1,2,3] J.-H. Chu,[1,2] J. G. Analytis,[1,2] Z. K. Liu,[1,2] K. Igarashi,[4] H.-H. Kuo,[1,2] X. L. Qi,[1,2] S. K. Mo,[3] R. G. Moore,[1] D. H. Lu,[1] M. Hashimoto,[2,3] T. Sasagawa,[4] S. C. Zhang,[1,2] I. R. Fisher,[1,2] Z. Hussain,[3] Z. X. Shen[1,2]*



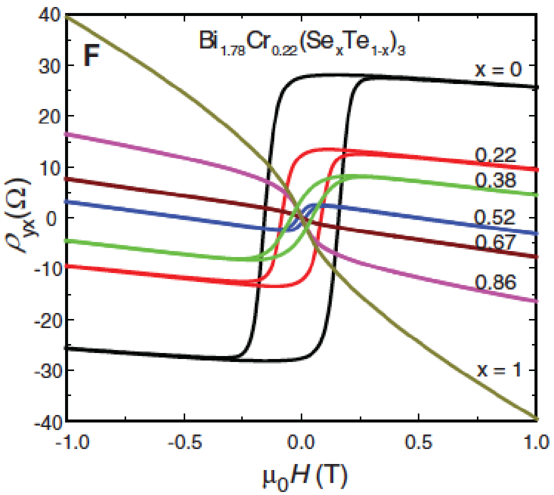
sive; indeed, we find that the Dirac gap can be observed in magnetically doped samples with or without bulk ferromagnetism (*19*). Furthermore, if E_F can be tuned into this surface-state gap, an insulating massive Dirac fermion state is formed;

Experiments of magnetic TIs

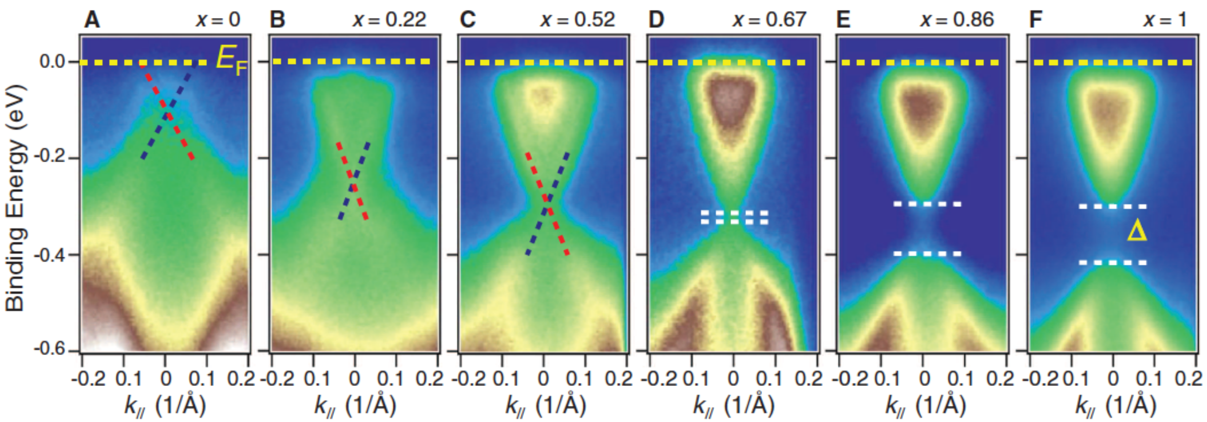
Science 339, 1582 (2013)

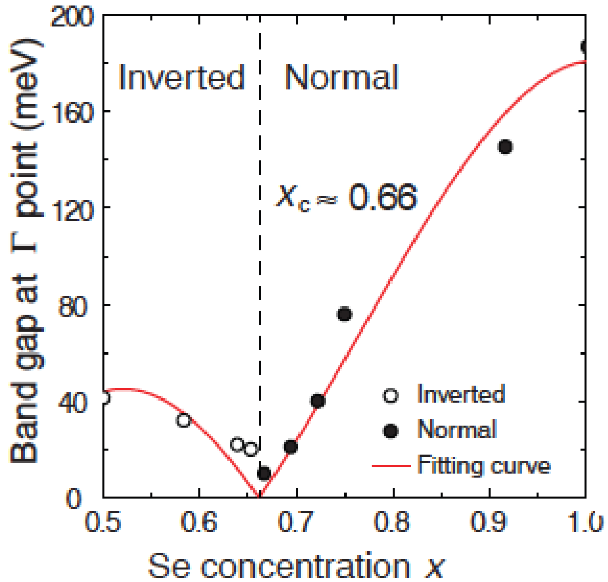
Topology-Driven Magnetic Quantum Phase Transition in Topological Insulators

Jinsong Zhang,[1*] Cui-Zu Chang,[1,2*] Peizhe Tang,[1*] Zuocheng Zhang,[1] Xiao Feng,[2] Kang Li,[2] Li-li Wang,[2] Xi Chen,[1] Chaoxing Liu,[3] Wenhui Duan,[1] Ke He,[2†] Qi-Kun Xue,[1,2] Xucun Ma,[2] Yayu Wang[1†]



$Bi_{2-y}Cr_y(Se_x\ Te_{1-x})_3$





Self-consistent theory

D. Kurebayashi
27pBF-3

$$H_{total} = H_e^{MF} + H_L^{MF} - N_i JMm$$

Electron

$$H_e^{MF} = \sum_{\boldsymbol{k}} c_{\boldsymbol{k}}^{+} \left[H_0(\boldsymbol{k}) + xJM\Sigma_z \right] c_{\boldsymbol{k}}$$

Σ_z : spin matrix

$$H_0(\boldsymbol{k}) = \sum_{i=1}^{3} R_i(\boldsymbol{k})\alpha_i + m_0(\boldsymbol{k})\alpha_4 + \varepsilon(\boldsymbol{k}) I$$

Bi_2Se_3 family Hexagonal lattice model

$$R_1(\mathbf{k}) = \frac{2}{\sqrt{3}} A_1 \sin\left(\frac{\sqrt{3}}{2}k_x\right)\cos\left(\frac{1}{2}k_y\right)$$

$$R_2(\mathbf{k}) = \frac{2}{3} A_1 \left[\cos\left(\frac{\sqrt{3}}{2}k_x\right)\sin\left(\frac{1}{2}k_y\right) + \sin\left(k_y\right)\right]$$

$$R_3(\mathbf{k}) = A_2 \sin(k_z)$$

$$m(\mathbf{k}) = m_0 - B_2\left[2 - 2\cos(k_z)\right] - \frac{4}{3}B_1\left[3 - 2\cos\left(\frac{\sqrt{3}}{2}k_x\right)\cos(\frac{1}{2}k_y) - \cos(ky)\right]$$

$$\varepsilon(\mathbf{k}) = -\epsilon_F + D_2\left[2 - 2\cos(k_z)\right] + \frac{4}{3}D_1\left[3 - 2\cos\left(\frac{\sqrt{3}}{2}k_x\right)\cos(\frac{1}{2}k_y) - \cos(ky)\right]$$

Self-consistent theory

D. Kurebayashi
27pBF-3

$$H_{total} = H_e^{MF} + H_L^{MF} - N_i JMm$$

Electron

$$H_e^{MF} = \sum_{\boldsymbol{k}} c_{\boldsymbol{k}}^{+} \left[H_0(\boldsymbol{k}) + xJM\Sigma_z \right] c_{\boldsymbol{k}}$$

Local spin

$$H_L^{MF} = Jm \sum_{l=1}^{N_{imp}} S_z(\boldsymbol{r}_l)$$

Self-consistent theory

D. Kurebayashi
27pBF-3

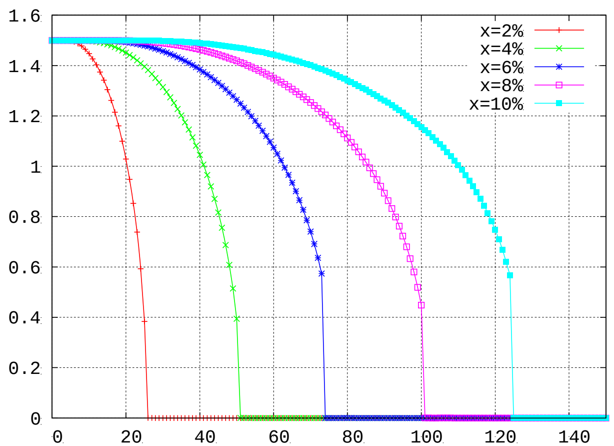
$$H_{total} = H_e^{MF} + H_L^{MF} - N_i JMm$$

<u>Electron</u>

$$H_e^{MF} = \sum_{\boldsymbol{k}} c_{\boldsymbol{k}}^{+} \left[H_0(\boldsymbol{k}) + xJM\Sigma_z \right] c_{\boldsymbol{k}}$$

<u>Local spin</u>

$$H_L^{MF} = Jm \sum_{l=1}^{N_{imp}} S_z(\boldsymbol{r}_l)$$



$$m = \frac{1}{N} \sum_{i=1}^{N} \langle c_i^{+} \Sigma_z c_i \rangle, \qquad M = \frac{1}{N_i} \sum_{l=1}^{N_{imp}} \langle S_z(\vec{R}_l) \rangle$$

Virtual crystal approximation

Self-consistent theory

D. Kurebayashi
27pBF-3

$$H_{total} = H_e^{MF} + H_L^{MF} - N_i JMm$$

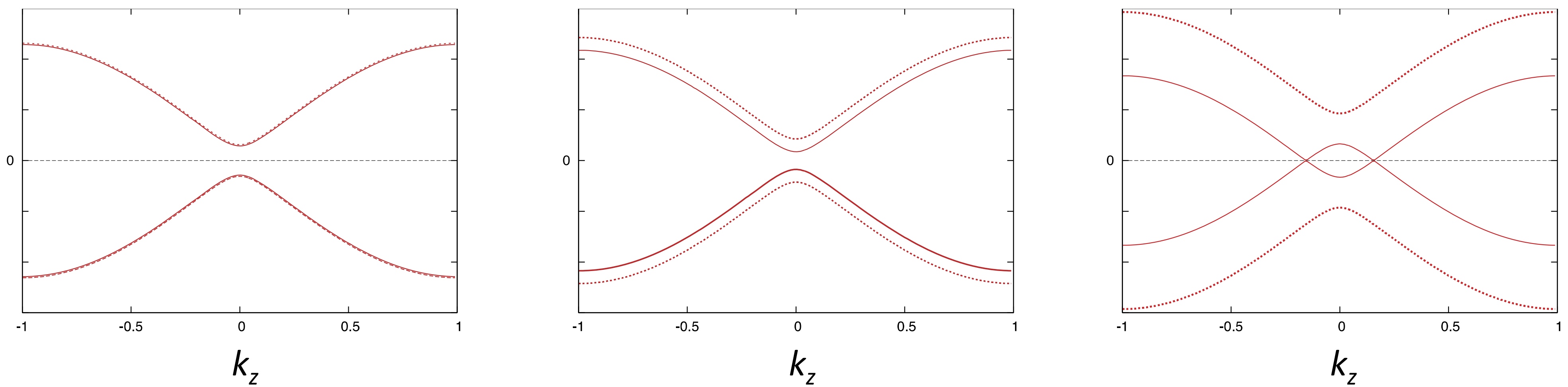
<u>Electron</u>

$$H_e^{MF} = \sum_{\boldsymbol{k}} c_{\boldsymbol{k}}^{+} \left[H_0(\boldsymbol{k}) + xJM\Sigma_z \right] c_{\boldsymbol{k}}$$

<u>Local spin</u>

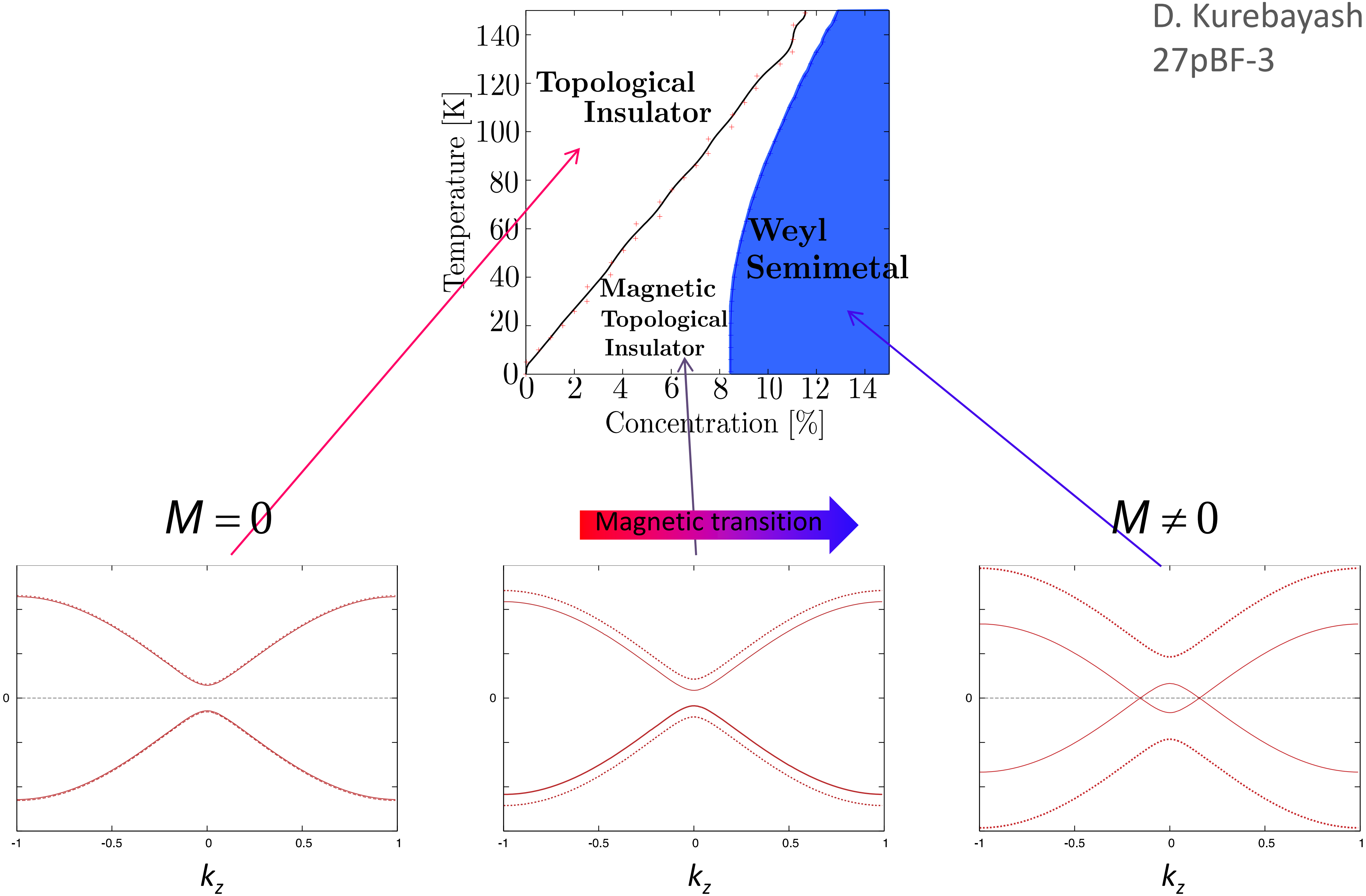
$$H_L^{MF} = Jm \sum_{l=1}^{N_{imp}} S_z(\boldsymbol{r}_l)$$

$M = 0$ Magnetic transition $M \neq 0$



Self-consistent theory

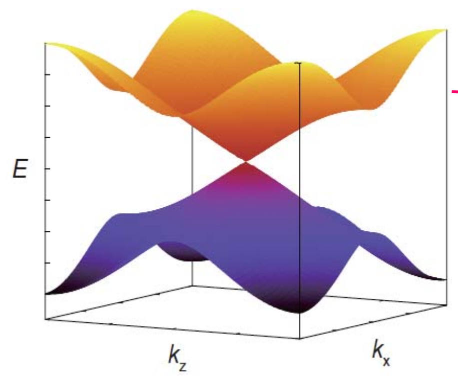
D. Kurebayashi
27pBF-3



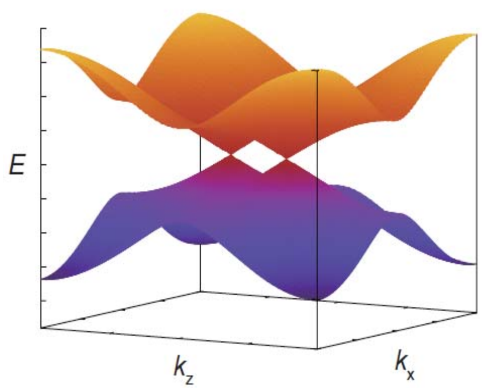
Phase diagram

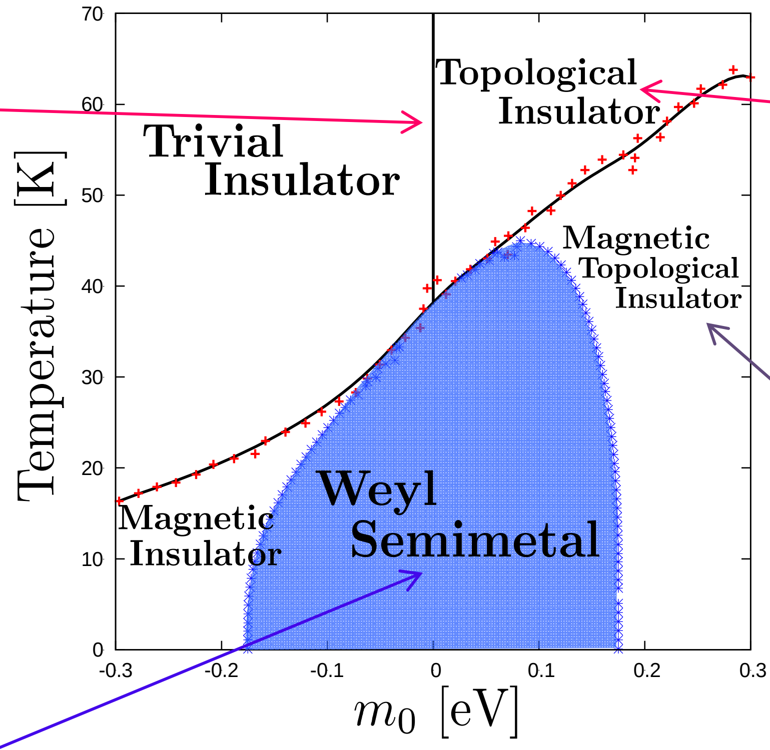
D. Kurebayashi
27pBF-3

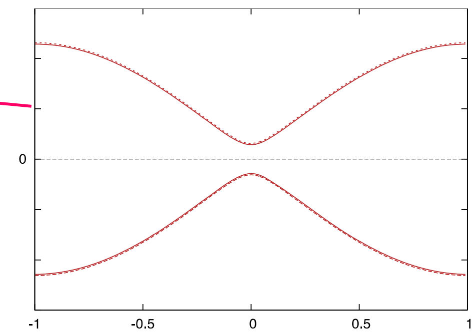
Dirac semimetals

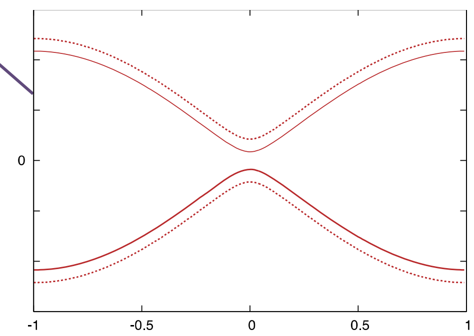


Weyl semimetals









weak SOC ⟷ strong SOC

日本物理学会　第69回年次大会
28pBF-3 チュートリアル講演

ゼロギャップ半導体における
スピン物性とワイル半金属相

outline

1. What is Weyl semimetal
2. How can it be realized
3. Charge transport
4. Novel phenomena

Charge transport of Weyl fermions

From 2d to 3d

topics

- Impurity scattering
- Quantum effect (localization)
- Anomalous Hall effect

Charge transport of Weyl fermions

From 2d to 3d

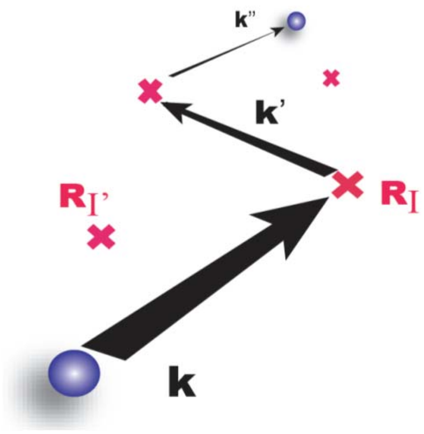
topics

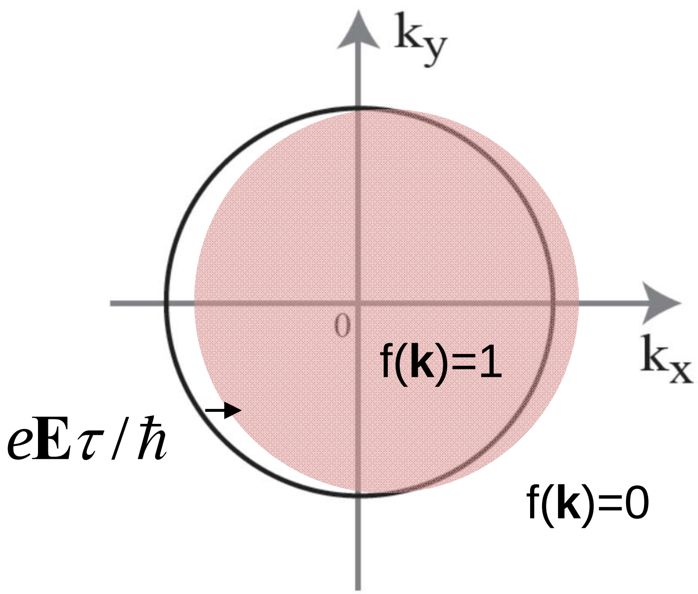
- Impurity scattering
- Quantum effect (localization)
- Anomalous Hall effect

Impurity scattering

$$\hbar/\tau = 2\pi \sum_{\boldsymbol{k}} \left|\langle \boldsymbol{k} \,|\, V \,|\, \boldsymbol{k}' \rangle\right|^2 (1 - \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{k}}')\,\delta(E_F - v_F |\boldsymbol{k}|)$$

τ : relaxation time





f(**k**) : distribution function in non-equilibrium

Impurity scattering

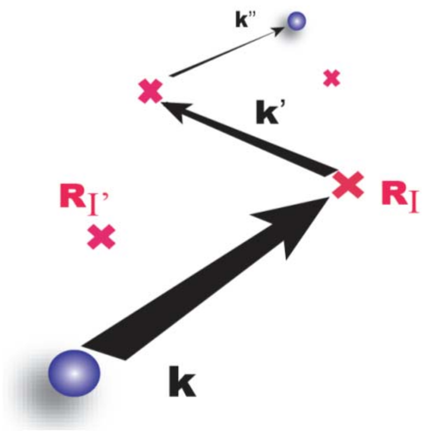
$$\hbar/_{\tau} = 2\pi \sum_{\boldsymbol{k}} \left|\langle \boldsymbol{k} \mid V \mid \boldsymbol{k}' \rangle\right|^2 (1 - \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{k}}')\delta(E_F - v_F \mid \boldsymbol{k} \mid)$$

Short-range scatterers

2d case

$$V(\boldsymbol{r}) = \sum_j u_j \delta(\boldsymbol{r} - \boldsymbol{R}_j)$$

τ : relaxation time



Impurity scattering

$$\hbar/\tau = 2\pi \sum_{\boldsymbol{k}} \left|\langle \boldsymbol{k} | V | \boldsymbol{k}' \rangle\right|^2 (1 - \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{k}}')\delta(E_F - v_F |\boldsymbol{k}|)$$

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$$V(\boldsymbol{r}) = \sum_j u_j \delta(\boldsymbol{r} - \boldsymbol{R}_j)$$

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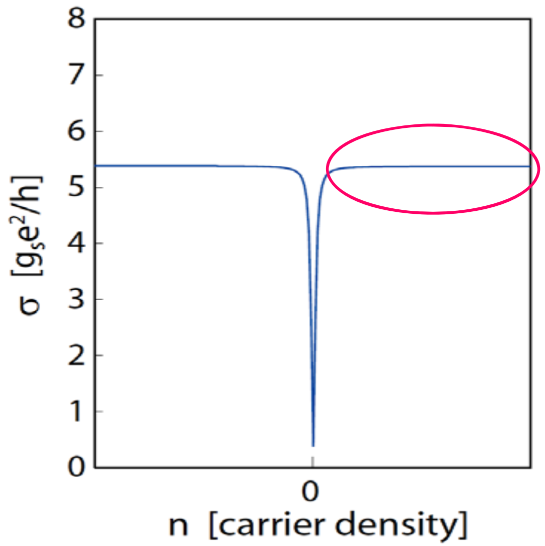
$$1/\tau_0 \propto \rho(E_F) \propto E_F$$

$$\sigma = \frac{2e^2}{h} E_F \tau \propto \frac{E_F}{E_F}$$



Shon & Ando (1998)

Impurity scattering

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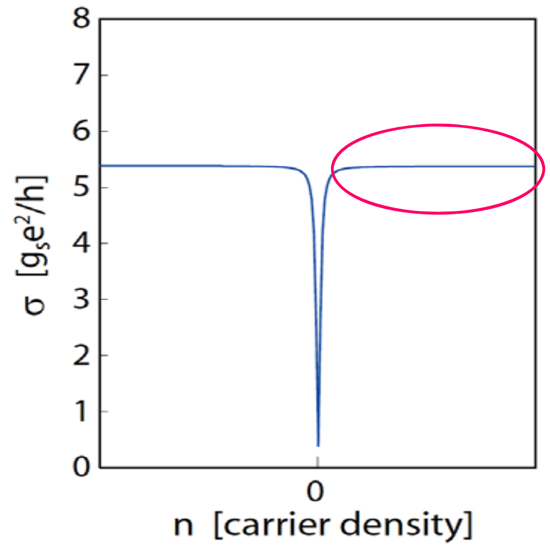


$$1/\tau_0 \propto \rho(E_F) \propto E_F$$

$$\sigma = \frac{2e^2}{h} E_F \tau \propto \frac{E_F}{E_F}$$

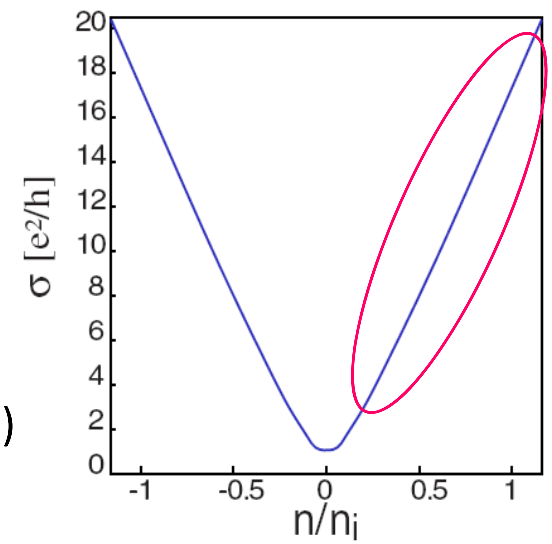
Coulomb scatterers

$$V(\boldsymbol{q}) = \sum_j \frac{2\pi e^2}{q^{d-1}} \exp(-i\boldsymbol{q}\cdot\boldsymbol{R}_j)$$

$$1/\tau_c \propto (1/k_F^2)\rho_F \propto 1/E_F$$

$$\sigma = \frac{2e^2}{h} E_F \tau \propto E_F^2 \propto k_F^2 \propto n$$

KN-MacDonald (2006)
Ando (2006)



Impurity scattering

$$\hbar/\tau = 2\pi \sum_{\boldsymbol{k}} \left|\langle \boldsymbol{k} | V | \boldsymbol{k}' \rangle\right|^2 (1 - \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{k}}')\delta(E_F - v_F |\boldsymbol{k}|)$$

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3d Weyl SM

$$1/\tau_0 \propto \rho(E_F) \propto E_F^2$$

$$\sigma \propto \frac{E_F^2}{E_F^2}$$

Coulomb scatterers

$$V(\boldsymbol{q}) = \sum_j \frac{2\pi e^2}{q^{d-1}} \exp(-i\boldsymbol{q} \cdot \boldsymbol{R}_j)$$

2d case

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$$\sigma = \frac{2e^2}{h} E_F \tau \propto E_F^2 \propto k_F^2 \propto n$$

3d Weyl SM

$$1/\tau_c \propto (1/k_F^4)\rho_F \propto 1/E_F^2$$

$$\sigma \propto E_F \propto n^{1/3}$$

Burkov-Hook-Balents (2011)

Charge transport of Weyl fermions

From 2d to 3d

topics

- Impurity scattering
- Quantum effect (localization)
- Anomalous Hall effect

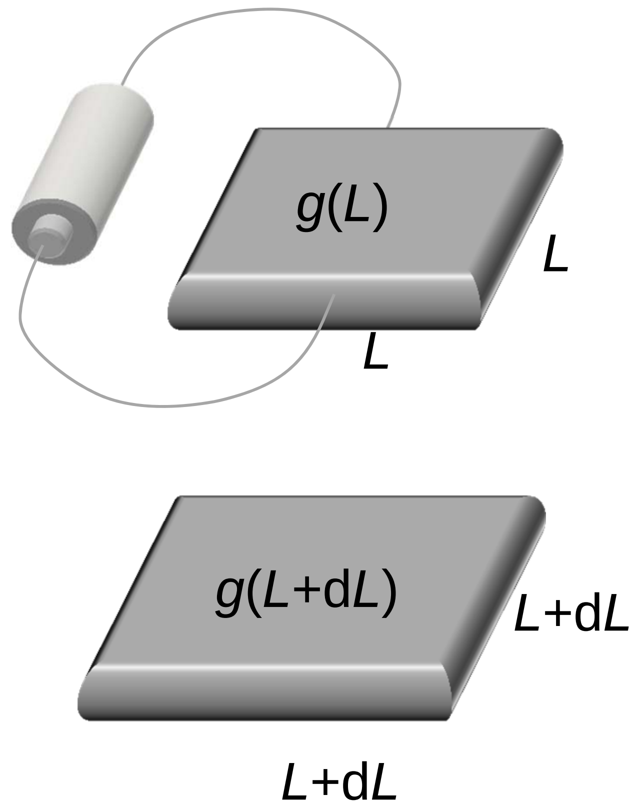
Anderson localization?

$$H = -i\vec{\sigma}\cdot\vec{\nabla} + V_0(\boldsymbol{x}) + \vec{\sigma}\cdot\vec{a}(\boldsymbol{x})$$

g: conductance

$$\frac{dg(L)}{dL} > 0 \longrightarrow \text{metal}$$

$$\frac{dg(L)}{dL} < 0 \longrightarrow \text{insulator}$$

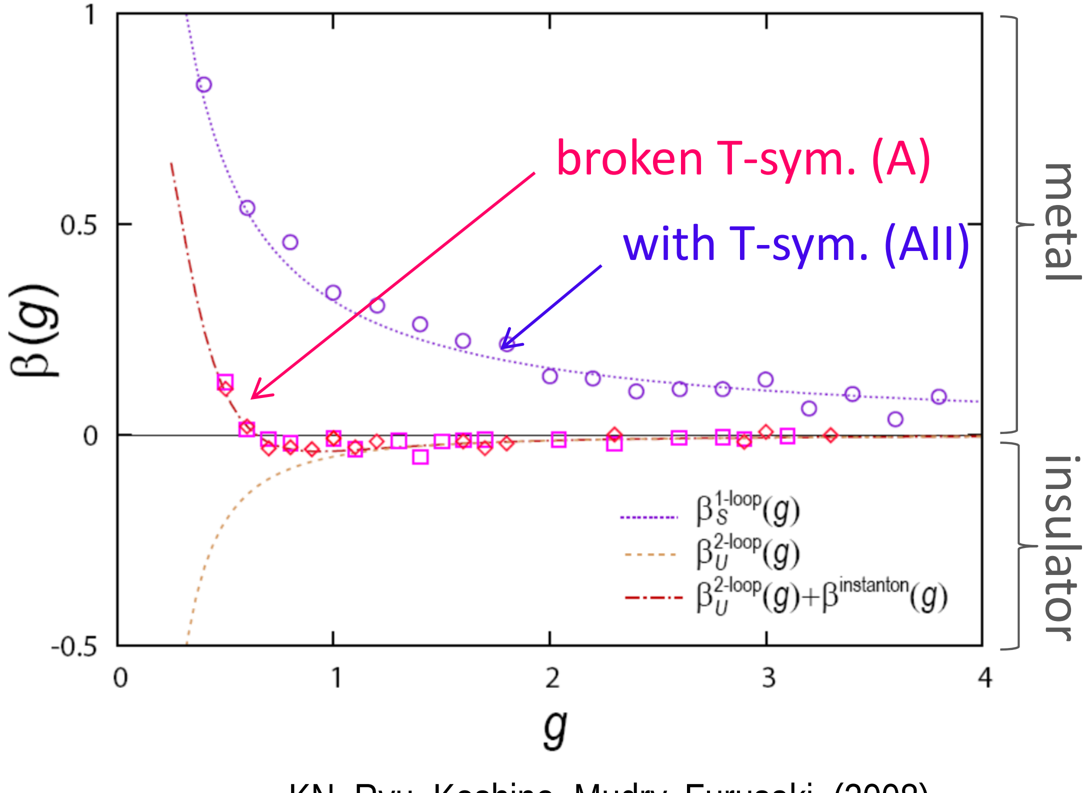


Anderson localization?

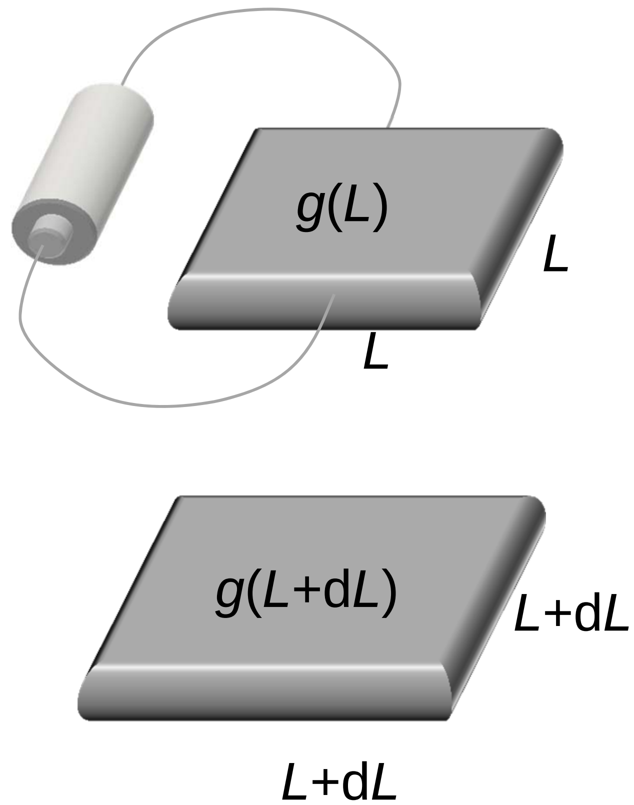
$$H = -i\vec{\sigma}\cdot\vec{\nabla} + V_0(\boldsymbol{x}) + \vec{\sigma}\cdot\vec{a}(\boldsymbol{x})$$

$$\beta(g) = \frac{L}{g}\frac{dg(L)}{dL}$$

2d case



KN, Ryu, Koshino, Mudry, Furusaki, (2008)



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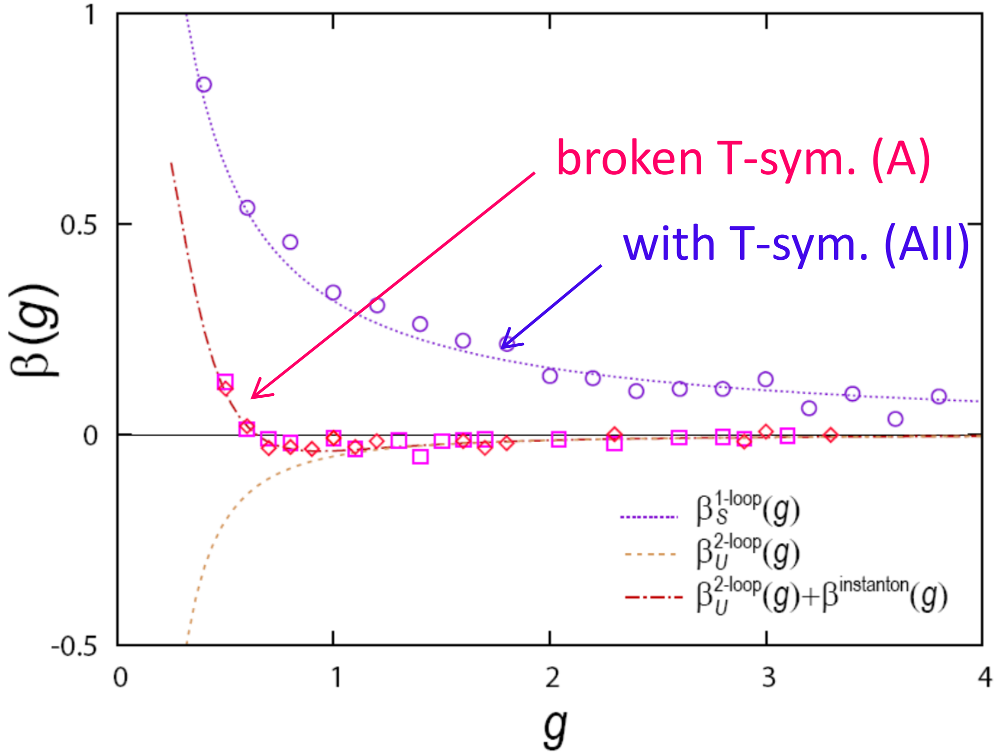
2d case



KN, Ryu, Koshino, Mudry, Furusaki, (2008)

- No numerical study has been done for 3d Weyl SM systems

-Effective field theory (class A)

2d case

$$S = \int d^2x\ \frac{g}{8}\mathrm{tr}\left[(\partial_\mu Q)^2\right] + S^{\text{topo}}_{\text{Prueskin}}$$

3d case

$$S = \int d^3x\ \frac{g}{8}\mathrm{tr}\left[(\partial_\mu Q)^2\right] + S^{\text{topo}}_{WZW}$$

Charge transport of Weyl fermions

From 2d to 3d

topics

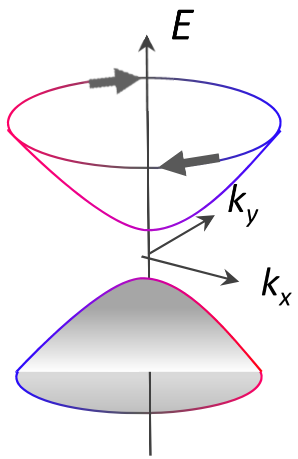
- Impurity scattering
- Quantum effect (localization)
- Anomalous Hall effect

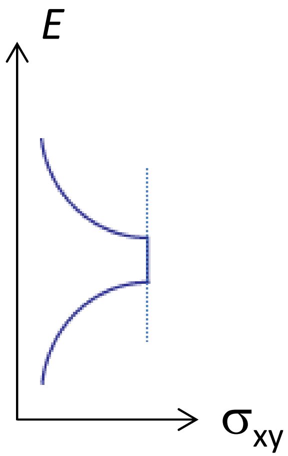
Anomalous Hall effect

AHE = Hall effect without external B field

2d case

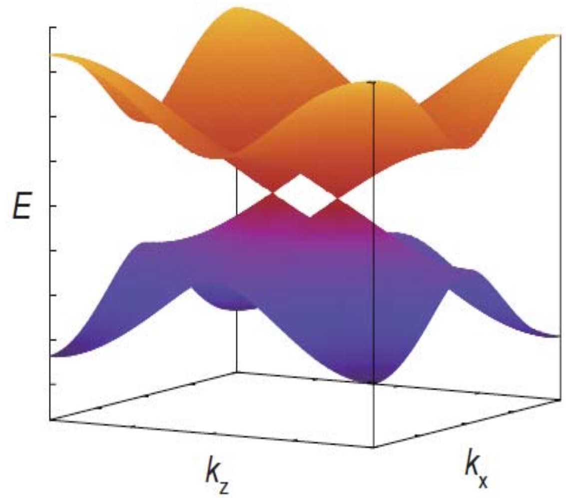
Massive Dirac -> AHE

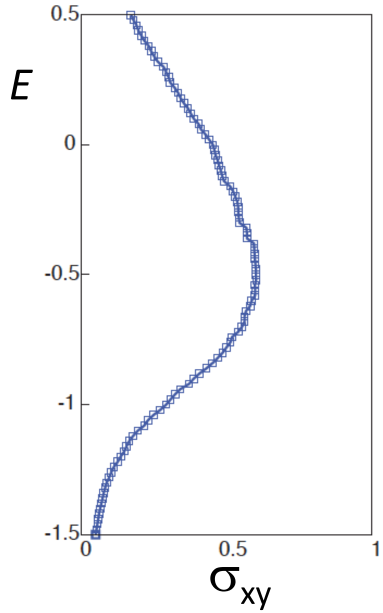




3d case

T-broken Weyl -> AHE



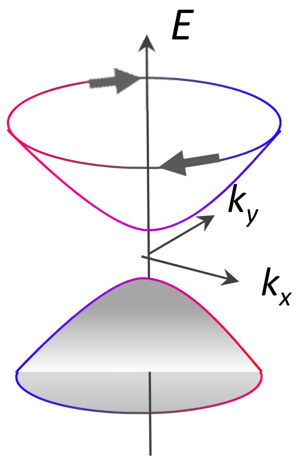


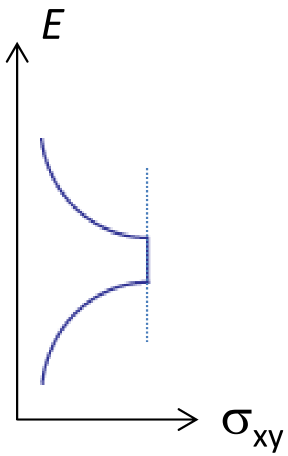
Anomalous Hall effect

$$H_{2d}(k_x,k_y) = \vec{\boldsymbol{R}}(k_x,k_y)\cdot\vec{\sigma}$$

2d case

Massive Dirac -> AHE



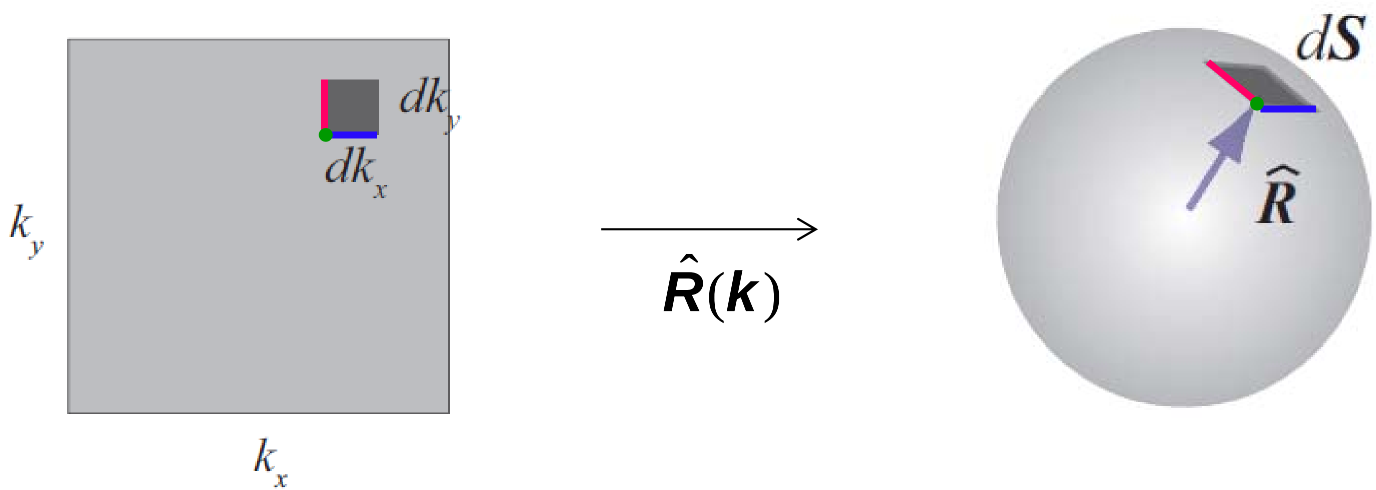


Anomalous Hall effect

$$H_{2d}(k_x, k_y) = \vec{\boldsymbol{R}}(k_x, k_y) \cdot \vec{\sigma}$$

$$\sigma_{xy} = -\frac{e^2}{h} \sum_{n,m,\vec{k}} f(E_{n\vec{k}}) 2\,\mathrm{Im} \frac{\langle u_{n\vec{k}} | v_x | u_{m\vec{k}} \rangle \langle u_{m\vec{k}} | v_y | u_{n\vec{k}} \rangle}{(E_{n\vec{k}} - E_{m\vec{k}})^2}$$
$$= -\frac{e^2}{h} \int \frac{d^2k}{4\pi} \hat{\boldsymbol{R}} \cdot \left(\frac{\partial \hat{\boldsymbol{R}}}{\partial k_x} \times \frac{\partial \hat{\boldsymbol{R}}}{\partial k_y} \right)$$

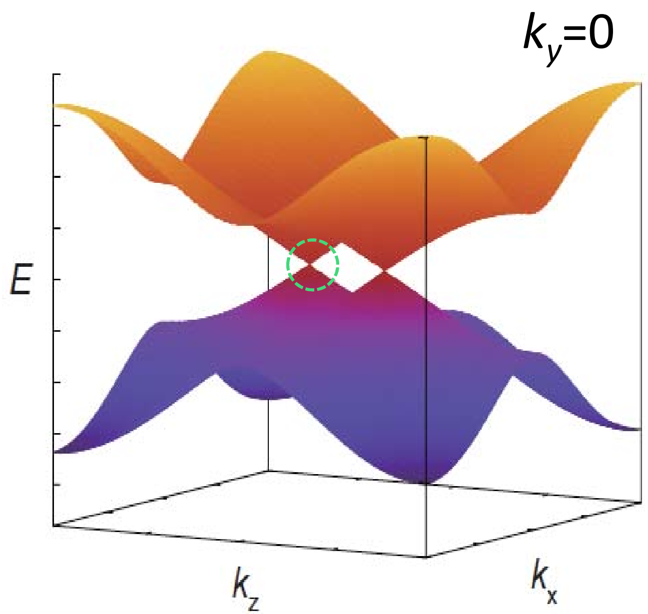
$$d\boldsymbol{S} = \left(\frac{\partial \hat{\boldsymbol{R}}}{\partial k_x} dk_x \right) \times \left(\frac{\partial \hat{\boldsymbol{R}}}{\partial k_y} dk_y \right)$$

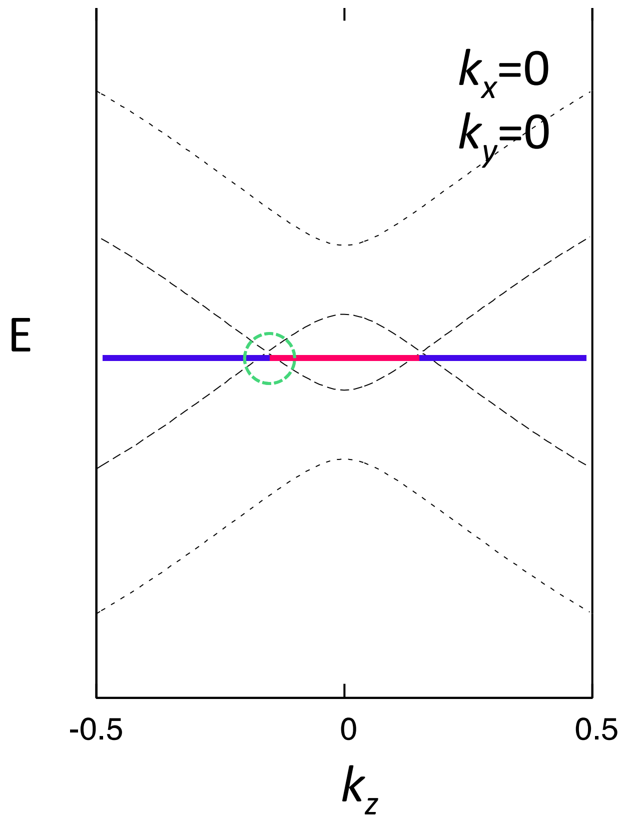


Anomalous Hall effect

$$H_{weyl}(k_x, k_y, k_z) = k_x\sigma_1 + k_y\sigma_2 + \Delta_m(k_z)\sigma_3$$

3d Weyl SM





Anomalous Hall effect

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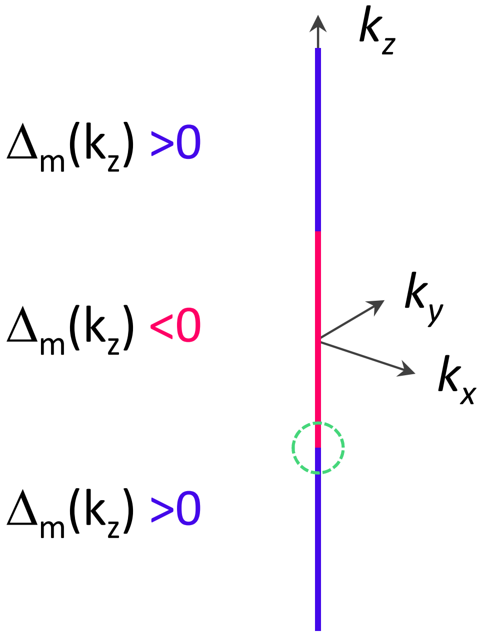
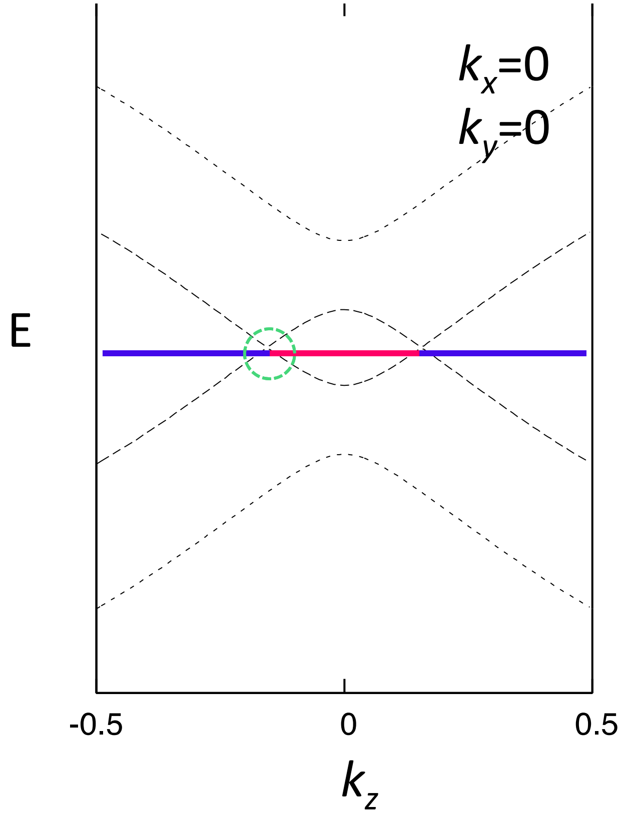
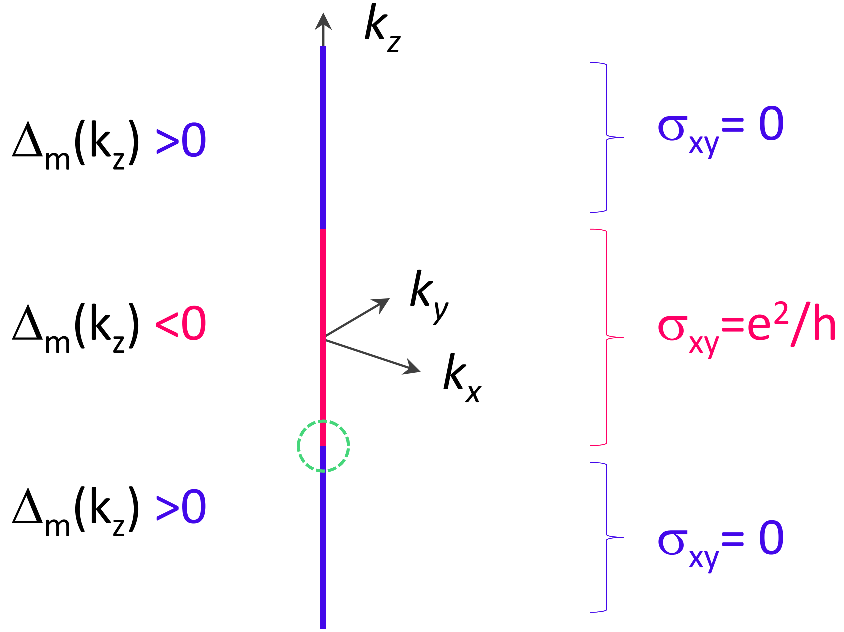




Anomalous Hall effect

$$H_{weyl}(k_x,k_y,k_z) = k_x\sigma_1 + k_y\sigma_2 + \Delta_m(k_z)\sigma_3$$

$$\sigma_{xy}^{2D}(k_z) = \frac{e^2}{2h}[1-\mathrm{sgn}(\Delta_m(k_z))]$$



$$\sigma_{xy}^{3D} = \int \frac{dk_z}{2\pi}\sigma_{xy}^{2D}(k_z)$$

$$= \frac{e^2}{h}2K_w$$

Anomalous Hall effect

$$H_{weyl}(k_x,k_y,k_z) = k_x\sigma_1 + k_y\sigma_2 + \Delta_m(k_z)\sigma_3$$

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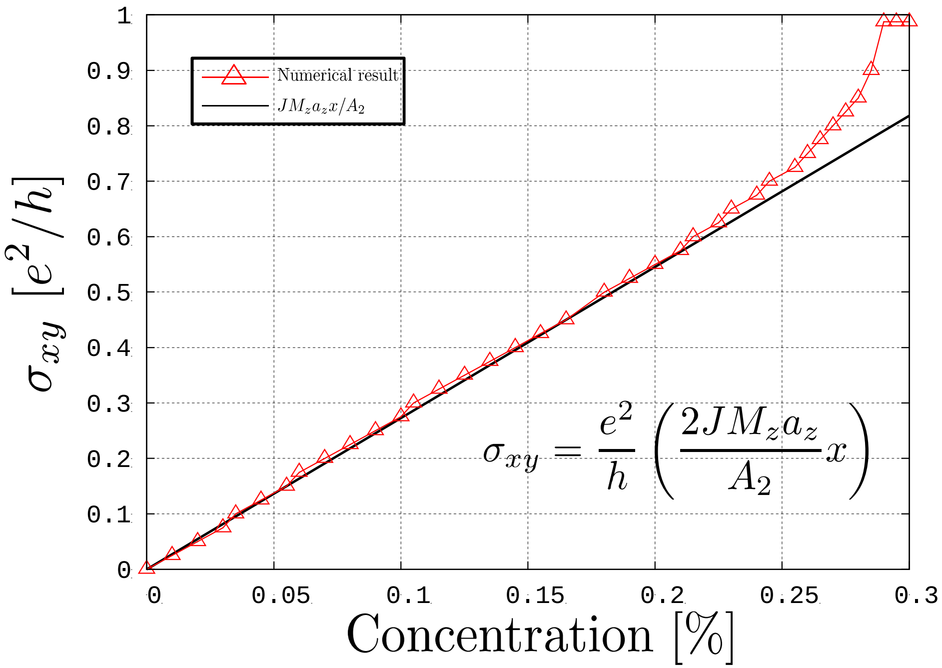
Kubo formula by D. Kurebayashi



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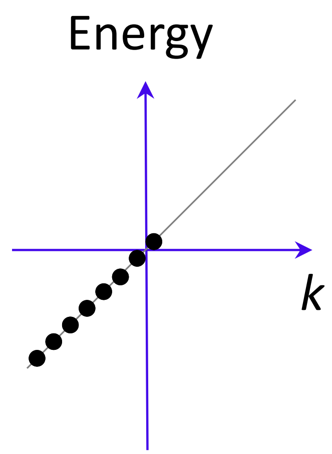
Chiral anomaly

Adler-Bell-Jackiw anomaly

1D Weyl fermions

$$H_{1D} = \int dx \; \psi^{+} \left[-i(\partial_x + ieA_x) \right] \psi$$



$$\frac{dN}{dt} = \int dx \frac{-e}{2\pi} E$$

N: particle number

3D Weyl fermions

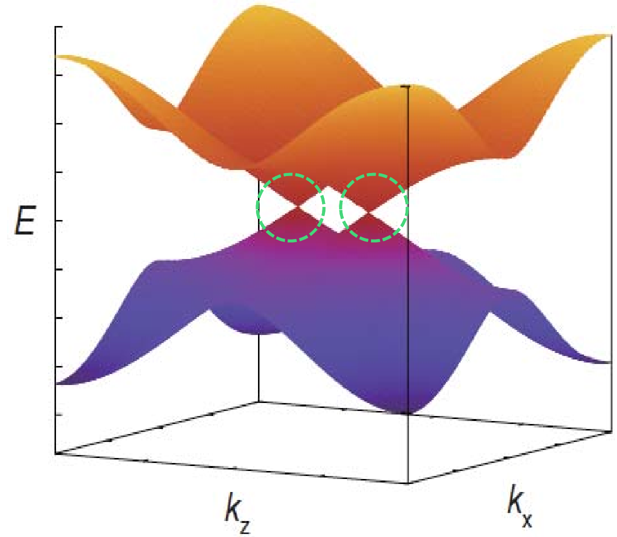
$$H_{3D} = \int d^3x \; \psi^{+} \left[-i\vec{\sigma} \cdot (\vec{\nabla} + ie\vec{A}) \right] \psi$$

$$\frac{dN}{dt} = \int d^3x \frac{e^2}{(2\pi)^2} \boldsymbol{E} \cdot \boldsymbol{B}$$

Chiral anomaly

3D Weyl semimetals

$$H = \begin{pmatrix} \vec{\sigma}\cdot(\vec{p}+e\vec{A}+\underline{J\vec{M}}) & 0 \\ 0 & -\vec{\sigma}\cdot(\vec{p}+e\vec{A}-\underline{J\vec{M}}) \end{pmatrix}$$



“chiral vector potential”

$$\partial_\mu j^\mu = \frac{e^2}{4\pi^2}\varepsilon^{\mu\nu\rho\lambda}F_{\mu\nu}\partial_\rho(\underline{JM}_\lambda)$$

$$= \partial_\mu\left(\frac{e^2}{4\pi^2}\varepsilon^{\mu\nu\rho\lambda}JM_\lambda F_{\rho\nu}\right)$$

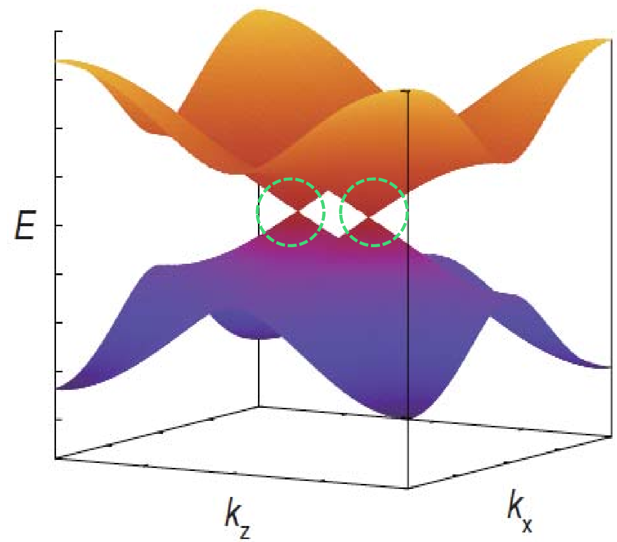
Zyusin & Burkov (2012)

Chiral anomaly

3D Weyl semimetals

$$H = \begin{pmatrix} \vec{\sigma}\cdot(\vec{p}+e\vec{A}+\underline{J\vec{M}}) & 0 \\ 0 & -\vec{\sigma}\cdot(\vec{p}+e\vec{A}-\underline{J\vec{M}}) \end{pmatrix}$$

"chiral vector potential"



$$\partial_\mu j^\mu = \frac{e^2}{4\pi^2}\varepsilon^{\mu\nu\rho\lambda}F_{\mu\nu}\partial_\rho(\underline{JM}_\lambda)$$

$$= \partial_\mu \underbrace{\left(\frac{e^2}{4\pi^2}\varepsilon^{\mu\nu\rho\lambda}JM_\lambda F_{\rho\nu}\right)}_{j^\mu_{AHE}}$$

$$\vec{j}_{AHE} = \frac{e^2}{2\pi h}J\vec{M}\times\vec{E}$$

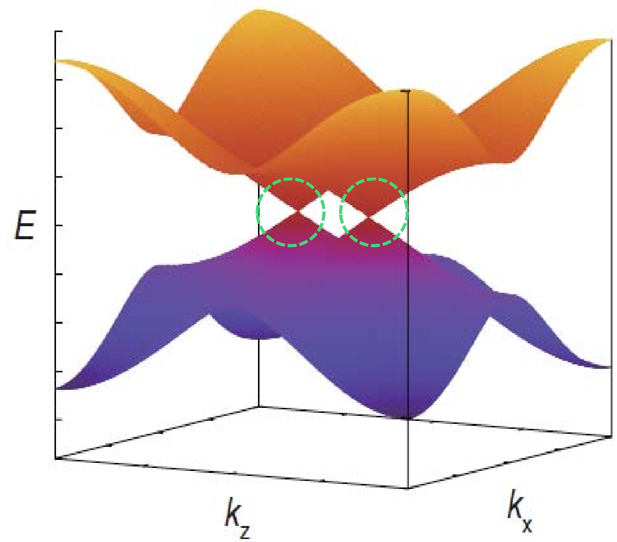
$$\rho_{AHE} = \frac{e^2}{2\pi h}J\vec{M}\cdot\vec{B}$$

Zyusin & Burkov (2012)

Chiral anomaly

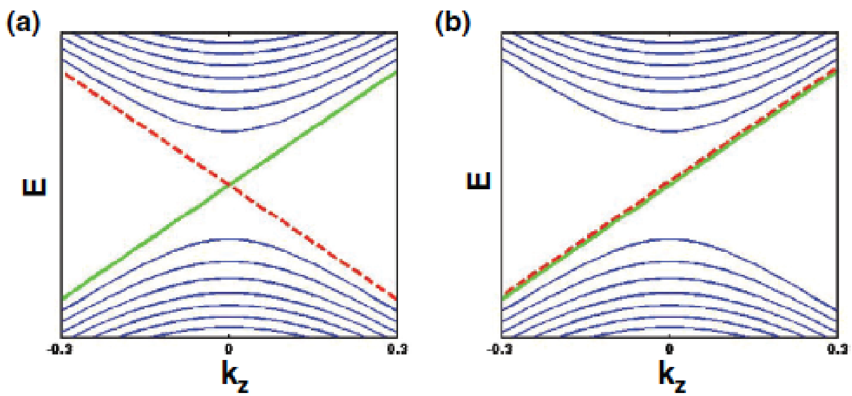
3D Weyl semimetals

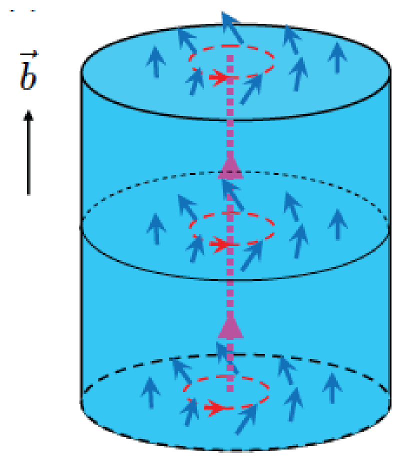
$$H = \begin{pmatrix} \vec{\sigma}\cdot(\vec{p}+e\vec{A}+\underline{J\vec{M}}) & 0 \\ 0 & -\vec{\sigma}\cdot(\vec{p}+e\vec{A}-\underline{J\vec{M}}) \end{pmatrix}$$



"chiral vector potential"

-> "chiral magnetic field"





Liu, Ye, Qi (2013)

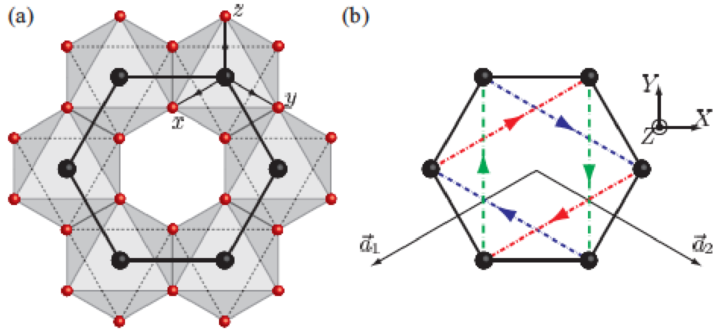
Interaction effects

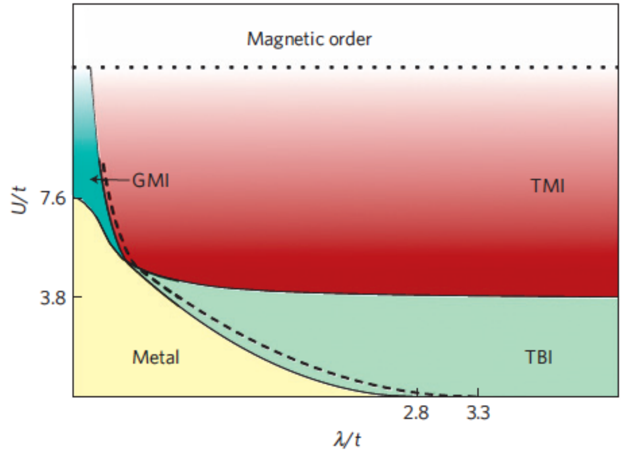
Topological Mott insulators

Raghu et. al. (2006)
Shitade et. al. (2008)

Pesin&Balents (2010)
Kurita, Yamaji, Imada (2011)
Yoshida et al. (2013)
Miyakoshi&Ohta (2013)

Na_2IrO_3

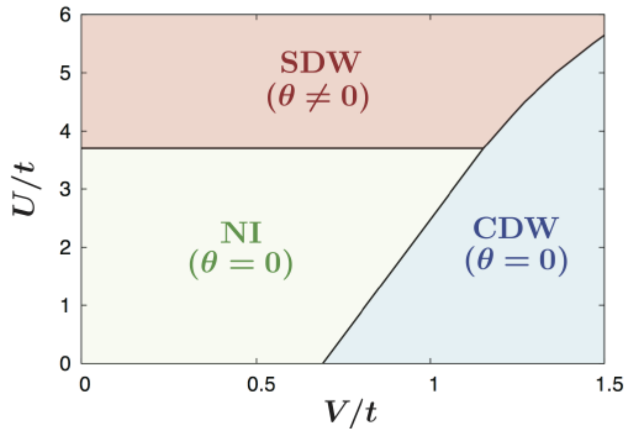




On-site U enhances SOC, causing a "topological Mott insulator" phase

Other ground states

Li, Wang, Qi, Zhang (2010)
Ooguri&Oshikawa (2012)
Sekine&KN (2014)



Axion Magneto-electric effect, Aoki phase (QCD analogy)

Interaction effects

p-electron systems

U: small -> long-range Coulomb ?

- Lattice gauge theory
- Strong coupling approach
- 16 order parameters

$$H = \sum_{i=1}^{3} p_i \alpha_i + m_0 \alpha_4 + b\Sigma_z$$

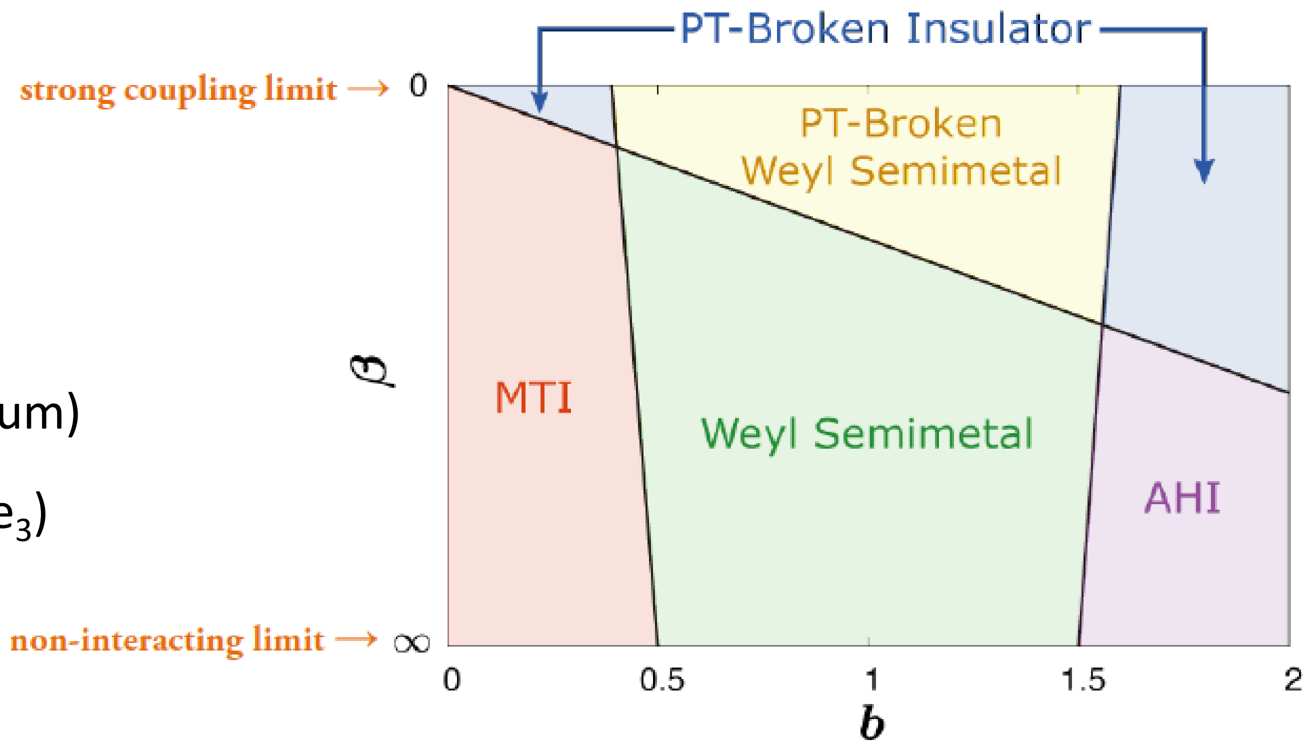
+ long-range Coulomb

Sekine, KN (2014)

$$\beta \equiv \frac{\varepsilon \hbar v_F}{e^2}$$

$\beta = 11$ (vacuum)

$\beta = 3$ (Bi_2Se_3)



Summary

Weyl semimetal is a novel gapless topological state

Weyl semimetal could be realized in Bi_2Se_3 family
(Cr-doped $Bi_2(Se_xTe_{1-x})_3$)

Novel quantum transport phenomena are expected
- Anderson (de)localization
- Anomalous Hall effect (chiral anomaly)

Effects of electron-electron interaction
- short range repulsion
- long range Coulomb interaction