日本物理学会2014年年次大会領域４チュートリアル講演

超伝導体におけるマヨラナ粒子
~実現と検出への展望~

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Outline

1. Introduction

(i) Basic features of Majorana fermions in topological superconductors

(ii) Possible realization

2. Why interesting
~ Exotic phenomena and possible experimental-detection scheme ~

✓ (i) Non-Abelian statistics

✓ (ii) Non-local correlation and “teleportation”

(iii) Majorana fermion as “fractionalization” of electron

(iv) Thermal responses

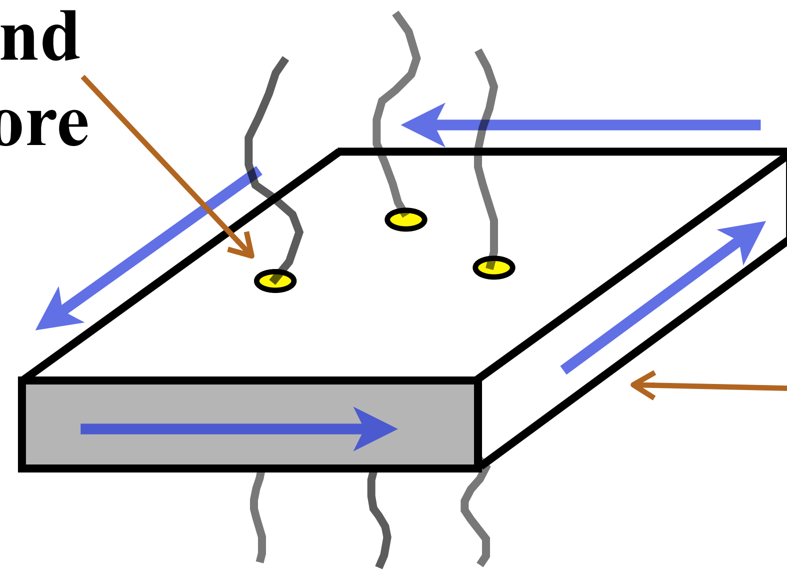
Majorana fermions in superconductors : Introduction

Majorana fermion : *particle = anti-particle !*

$$\gamma^\dagger = \gamma$$

Ettore Majorana

c.f. neutrino ?



Majorana fermion in SC: equal-weight superposition of electron and hole

Bogoliubov quasiparticle $\gamma^\dagger = \int d\boldsymbol{r}[u_E(\boldsymbol{r})c^\dagger(\boldsymbol{r}) + v_E(\boldsymbol{r})c(\boldsymbol{r})]$

Spinless p+ip SC

Bogoliubov quasiparticle $\gamma^\dagger = \int d\boldsymbol{r}[u_E(\boldsymbol{r})c^\dagger(\boldsymbol{r}) + v_E(\boldsymbol{r})c(\boldsymbol{r})]$

Because of p-h symmetry of BCS Hamiltonian

$$\Gamma\hat{\mathcal{H}}\Gamma^{-1} = -\hat{\mathcal{H}}^* \qquad \Gamma = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

N.B. $\Gamma^2 = 1$

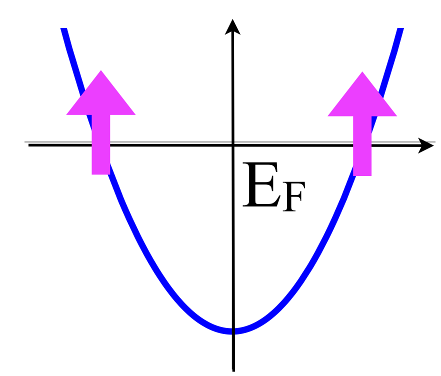
if $\hat{\mathcal{H}}\phi = E\phi$ then $\hat{\mathcal{H}}\Gamma\phi^* = -E\Gamma\phi^*$ $\qquad \phi^T = (u, v)$

→ **If there is only one independent zero energy solution of BdG eq.**

$$\phi = \Gamma\phi^* \rightarrow u_0^* = v_0 \rightarrow \gamma^\dagger = \gamma$$

Non-degenerate zero energy Bogoliubov quasiparticle is Majorana !!
equal-weight superposition of electron and hole !!

This argument also applies to spin-triplet SC, spin-singlet with SO int. etc. (class D, DIII, BDI)



How about the case that zero-energy modes are degenerate ?

class C, CI, (spin-singlet SC)

SU(2) spin symmetry implies two degenerate zero-energy states, if they exist

$$\gamma_1 = \int dr[u_\uparrow(r)c_\uparrow(r) - v_\downarrow(r)c_\downarrow^\dagger(r)] \qquad \gamma_2 = \int dr[u_\downarrow(r)c_\downarrow(r) + v_\uparrow(r)c_\uparrow^\dagger(r)]$$

$$\phi_1^T = (u_\uparrow, 0, 0, -v_\downarrow) \qquad \phi_2^T = (0, u_\downarrow, v_\uparrow, 0)$$

($\phi^T = (u_\uparrow, u_\downarrow, v_\uparrow, -v_\downarrow)$)

p-h conjugate

$$(\Gamma\phi_1)^{*T} = (0, iv_\downarrow^*, -iu_\uparrow^*, 0)$$

p-h conjugate

$$(\Gamma\phi_2)^{*T} = (-iv_\uparrow^*, 0, 0, -iu_\downarrow^*)$$

$$\hat{\mathcal{H}}\phi = E\phi$$

$$\hat{\mathcal{H}}(\Gamma\phi)^* = -E(\Gamma\phi)^*$$

$$\Gamma = i\begin{pmatrix} 0 & 1_{2\times2} \\ 1_{2\times2} & 0 \end{pmatrix}$$

N.B. $\Gamma^2 = -1$

If there are only two zero-energy modes,

$$\rightarrow \phi_1 = (\Gamma\phi_2)^* \qquad \phi_2 = (\Gamma\phi_1)^*$$

$$\rightarrow u_\uparrow = -iv_\uparrow^* \qquad \gamma_1^\dagger = i\gamma_2$$

$$u_\downarrow = iv_\downarrow^* \qquad \gamma_2^\dagger = i\gamma_1$$

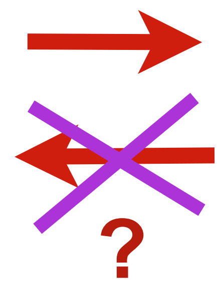
$$\tilde{\gamma}_1 = \frac{1}{2}(\gamma_1 + i\gamma_2) \qquad \tilde{\gamma}_2 = \frac{1}{2i}(\gamma_1 - i\gamma_2) \quad \Rightarrow \quad \tilde{\gamma}_1^\dagger = \tilde{\gamma}_1 \qquad \tilde{\gamma}_2^\dagger = \tilde{\gamma}_2$$

Majorana !!

equal-weight superposition of electron and hole !!

$$\gamma^\dagger = \int d\boldsymbol{r}[u_E(\boldsymbol{r})c^\dagger(\boldsymbol{r}) + v_E(\boldsymbol{r})c(\boldsymbol{r})]$$

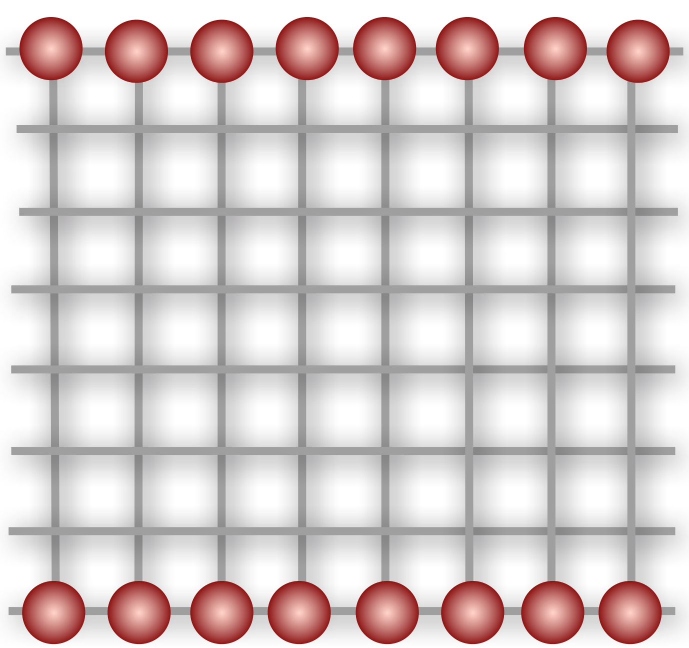
zero-energy Bogoliubov quasiparticle in SC



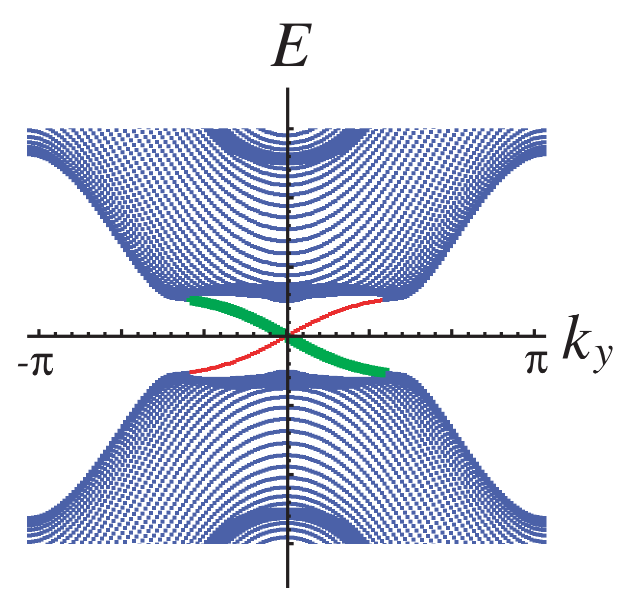
equal-weight superposition of electron and hole

$$|u| = |v|$$

Majorana edge state



2D top. SC



Majorana fermions with nonzero energy satisfy

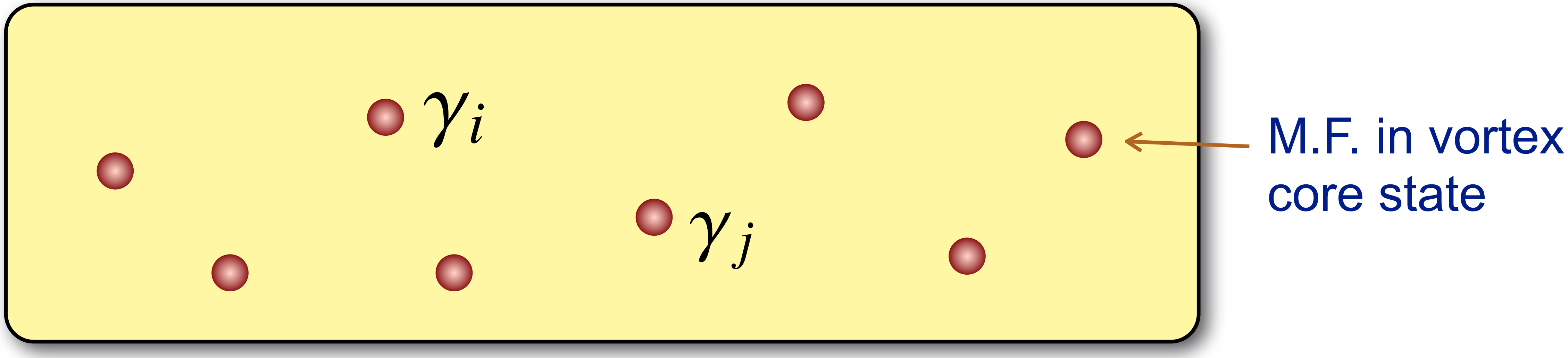
$$|u| = |v|$$

Majorana condition for nonzero energy states

$$\gamma_k^\dagger = \gamma_{-k}$$

Bogoliubov q.p. with $|u| = |v|$ ***is Majorana***

Anti-commutation relation of Majorana fields holds



$$\gamma_i^2 = 1,$$

$$\gamma_i \gamma_j = -\gamma_j \gamma_i \quad \text{for} \quad i \neq j.$$

follow from

$$\gamma_i = \sqrt{2} \sum_\sigma \int d\boldsymbol{r} [u_{i\sigma}(\boldsymbol{r}) \psi_\sigma(\boldsymbol{r}) + u_{i\sigma}^*(\boldsymbol{r}) \psi_\sigma^\dagger(\boldsymbol{r})]$$

$$\{\psi(\boldsymbol{r}), \psi(\boldsymbol{r}')\} = 0 \qquad \{\psi(\boldsymbol{r}), \psi^\dagger(\boldsymbol{r}')\} = \delta(\boldsymbol{r} - \boldsymbol{r}')$$

Topology and Majorana Fermion

(Schnyder, Ryu, Furusaki, Ludwig; Kitaev; Teo, Kane)

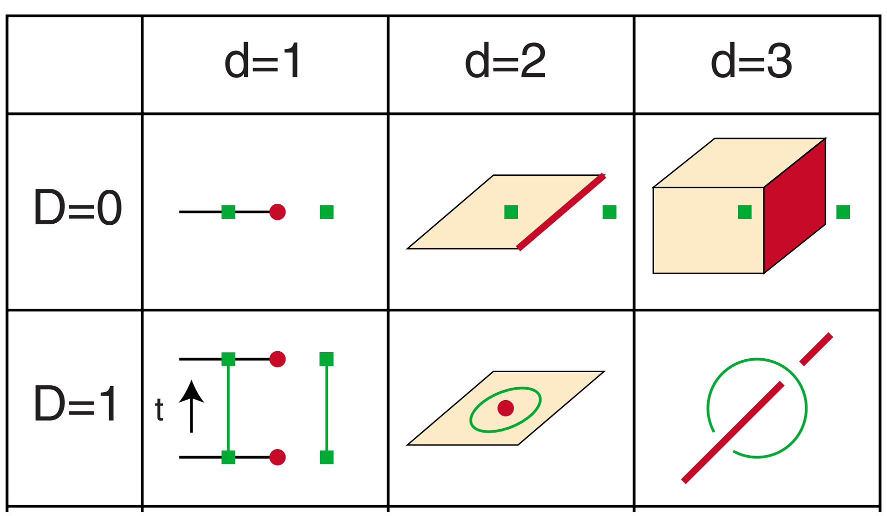
Θ TRS

Ξ PHS

vortex of 2D top. SC

edge of 2D top. SC

	Symmetry			$\delta = d - D$			
	AZ	Θ^2	Ξ^2	0	1	2	3
odd parity	BDI	1	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
	D	0	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
	DIII	−1	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
even parity	C	0	−1	0	0	$2\mathbb{Z}$	0
	CI	1	−1	0	0	0	$2\mathbb{Z}$



vortex of even-parity top. SC

vortex of 3D trivial (or weak top.) SC (e.g. URu_2Si_2 ?)

odd-parity top. SCs may be advantageous for realizing and detecting topologically protected Majorana fermions.

crystalline symmetry (mirror, mirror +TRS, etc.) provides topological protection of M.F. even for trivial classes above

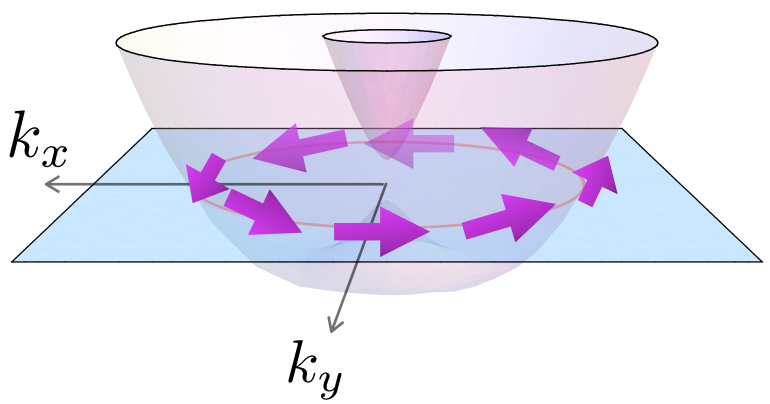
(Fang, Gilbert, Bernevig; Shiozaki, Sato)

Possible realization

2D class D with broken TRS

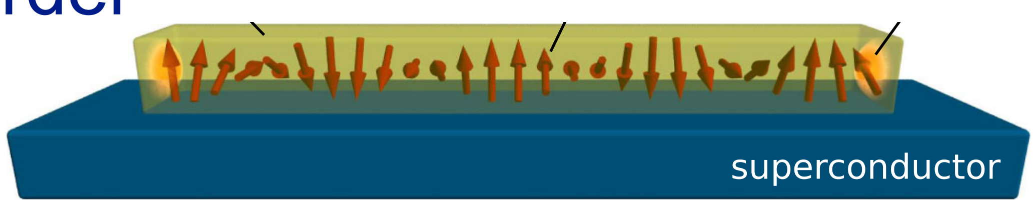
- Sr_2RuO_4 *(Maeno et al.)*

- spin-singlet SC with strong spin-orbit interaction and Zeeman fields
  (Sato,Takahashi,S.F.; Sau et al.; Alicea; Luchyn et al.; Oreg et al.)



- Proximity-induced SC on surface of top. insulator *(L. Fu,C. L. Kane)*
  (TRS must be broken by magnetic fields)

- spin-singlet SC coupled with spiral magnetic order
  (Braunecker, Simon; Klinovaja et al.; Vazifeh, Franz; Nakosai,Tanaka,Nagaosa)



class DIII with TRS

- Helium 3, B phase

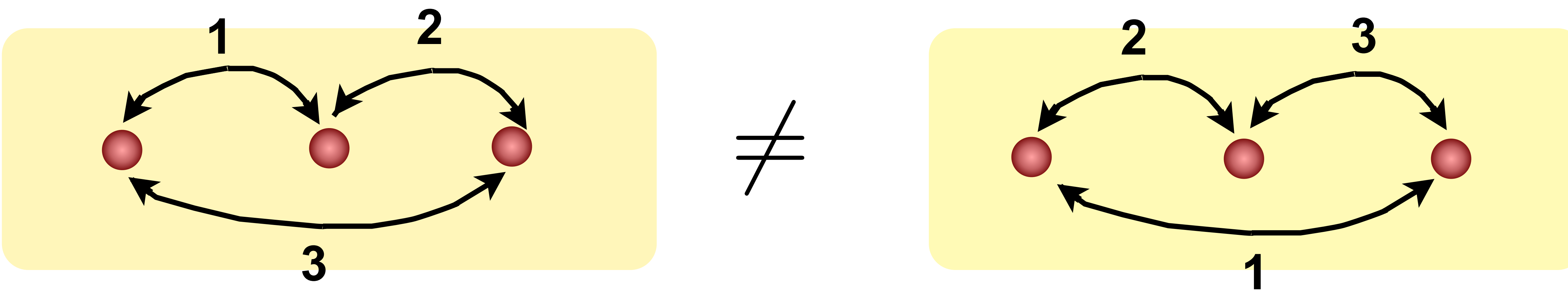
- $Cu_xBi_2Se_3$ *(Fu, Berg; Sasaki et al.)*

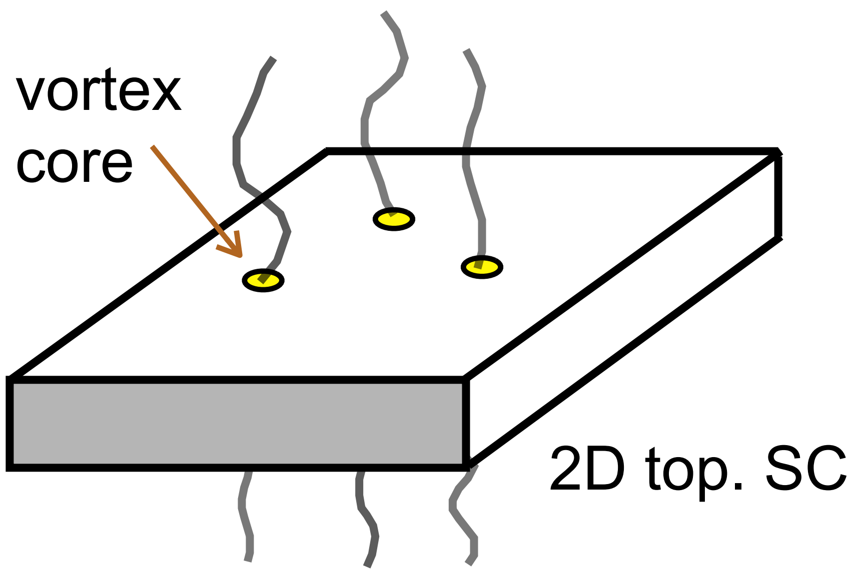
Why interesting
~ Exotic phenomena
and possible detection scheme ~

Non-Abelian Statistics

Non-Abelian statistics

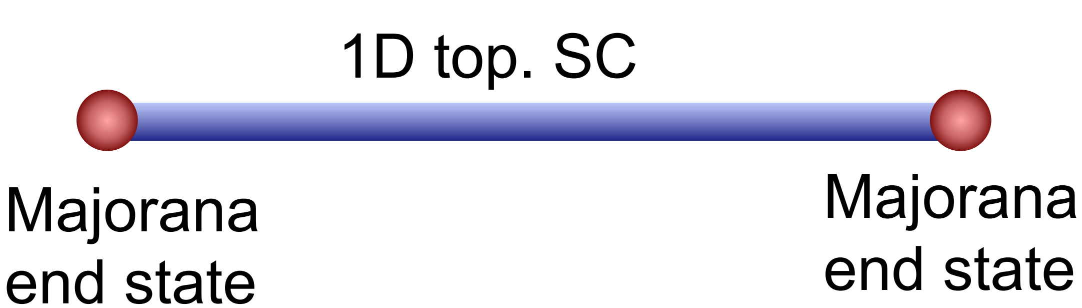
exchange (braiding) of particles is non-commutative !!





- Originally proposed for vortex core states *(Read-Moore, Read-Green, Ivanov)*

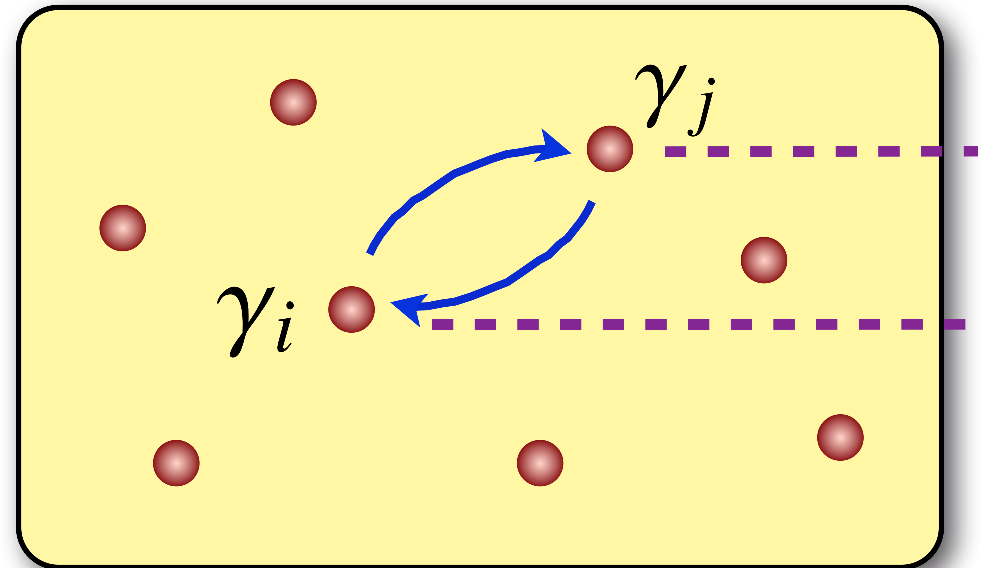
- Edge states *without vortices* also obey non-Abelian statistics



General properties of Majorana zero-energy state in topological SC.

Exchange (braiding) operation of Majorana zero modes

$$\gamma_i \rightarrow \gamma_j, \quad \gamma_j \rightarrow -\gamma_i$$



Sign change is not due to vortex, but due to Fermion-parity conservation !!

(Clarke, Sau, Tewari; Halperin et al.)

U_{12} **: unitary operation for braiding of** γ_1 **and** γ_2

$$s_2\gamma_2 = U_{12}\gamma_1 U_{12}^{\dagger} \qquad s_1\gamma_1 = U_{12}\gamma_2 U_{12}^{\dagger}$$

(G.S. is separated from excited states by finite energy gap)

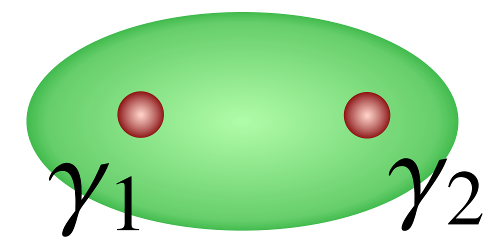
$s_1 \quad s_2$: phase factor

From $s_1^2\gamma_1^2 = s_2^2\gamma_2^2 = 1$ and $\gamma_1^2 = 1$, $\gamma_2^2 = 1$ → $s_1 = \pm 1 \quad s_2 = \pm 1$

$$s_2\gamma_2 = U_{12}\gamma_1 U_{12}^\dagger \qquad s_1\gamma_1 = U_{12}\gamma_2 U_{12}^\dagger \qquad \begin{aligned} s_1 &= \pm 1 \\ s_2 &= \pm 1 \end{aligned}$$

How the occupation number of complex fermion $\psi_{12} = (\gamma_1 + i\gamma_2)/2$ is changed by braiding of Majorana zero modes ?

$$n_{12} = \psi_{12}^\dagger \psi_{12} = \frac{1}{2}(1 + i\gamma_1\gamma_2)$$



$$U_{12} n_{12} U_{12}^\dagger = \frac{1}{2} + \frac{i}{2} U_{12}\gamma_1\gamma_2 U_{12}^\dagger = \frac{1}{2} - \frac{i}{2} s_1 s_2 \gamma_1\gamma_2$$

If γ_1 and γ_2 are sufficiently far from other Majorana fermions,

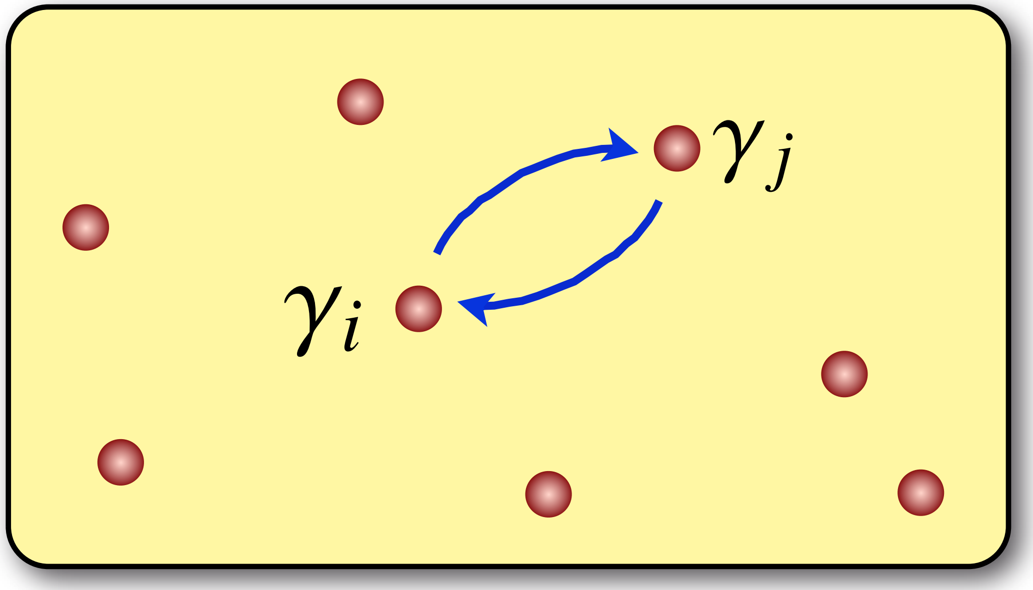
Fermion-parity of n_{12} is not changed by U_{12}

$$s_1 s_2 = -1$$

$$\gamma_1 \to \gamma_2, \qquad \gamma_2 \to -\gamma_1$$

non-Abelian statistics even without vortices

Braiding rule of Majorana zero modes :



$$\gamma_i \rightarrow \gamma_j, \quad \gamma_j \rightarrow -\gamma_i$$

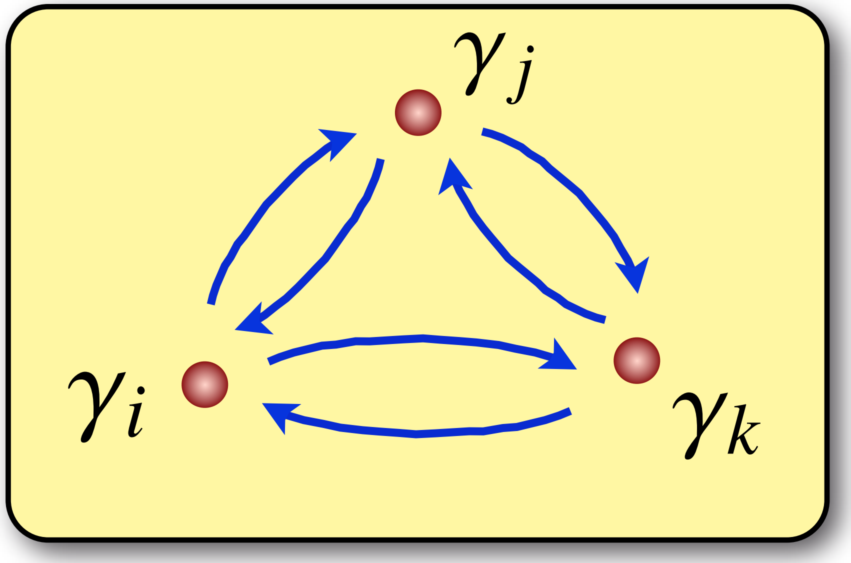
Exchange (braiding) operator

$$U_{ij} = \exp\left(-\frac{\pi}{4}\gamma_i\gamma_j\right) = \frac{1}{\sqrt{2}}(1 - \gamma_i\gamma_j)$$

$$U_{ij}\gamma_i U_{ij}^\dagger = \gamma_j \qquad U_{ij}\gamma_j U_{ij}^\dagger = -\gamma_i$$

Non-commutativity of exchange (braiding) operation

$$U_{ij}U_{jk} - U_{jk}U_{ij} = -\gamma_i\gamma_k = i(2n_{ik} - 1) \neq 0$$



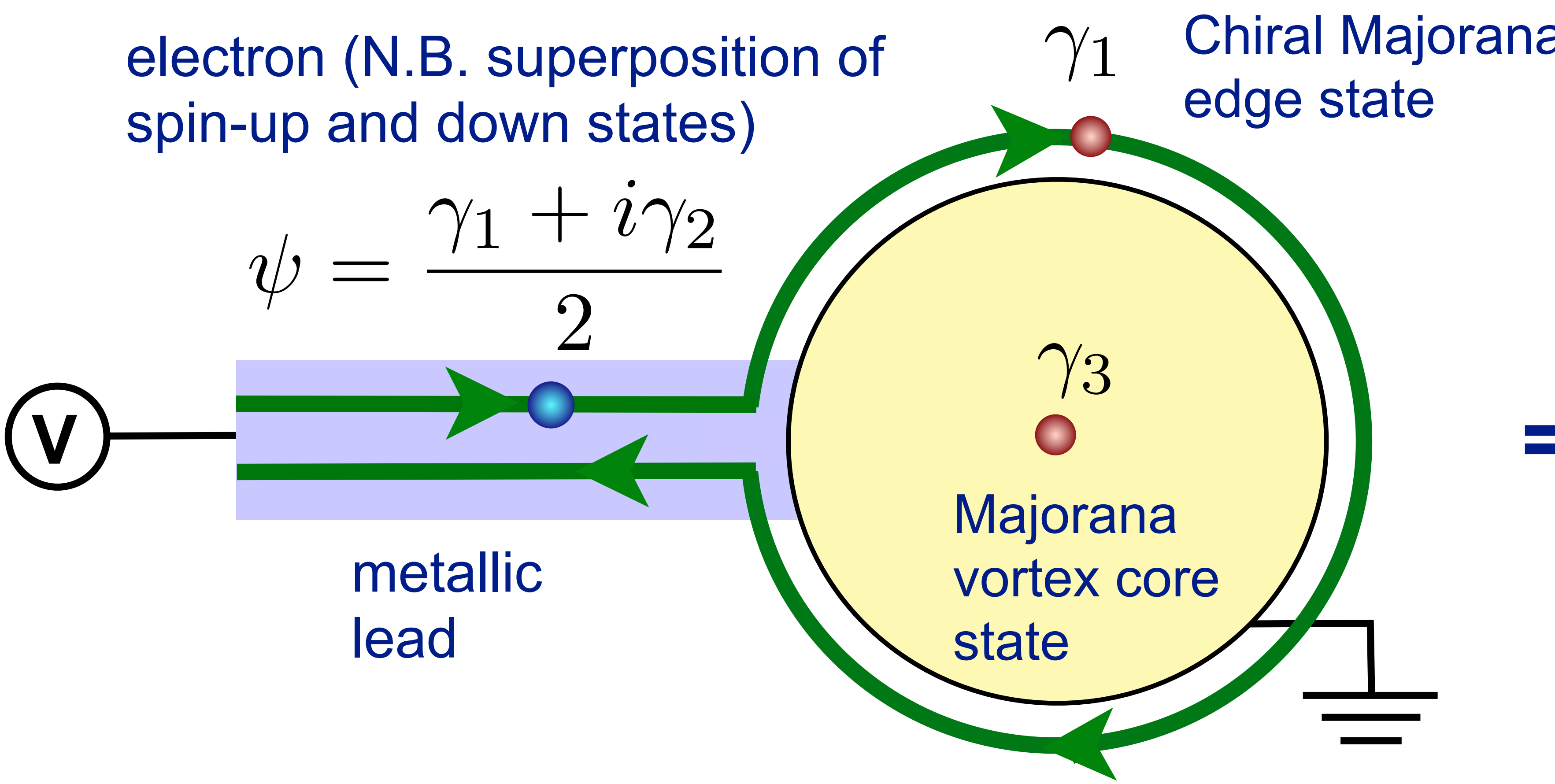
Non-Abelian character !

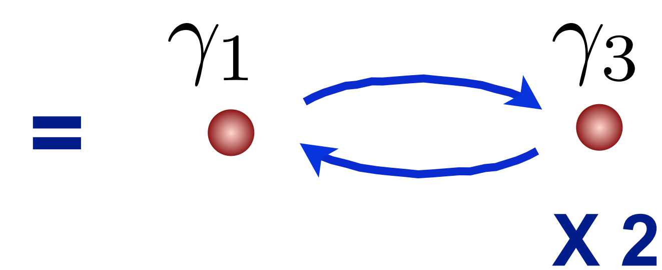
How to detect Non-Abelian statistics ? (2D class D top. SC)

Interferometer experiment I

one-lead conductance measurement

(Law, Lee, Ng; Li, Fleury, Buttiker)





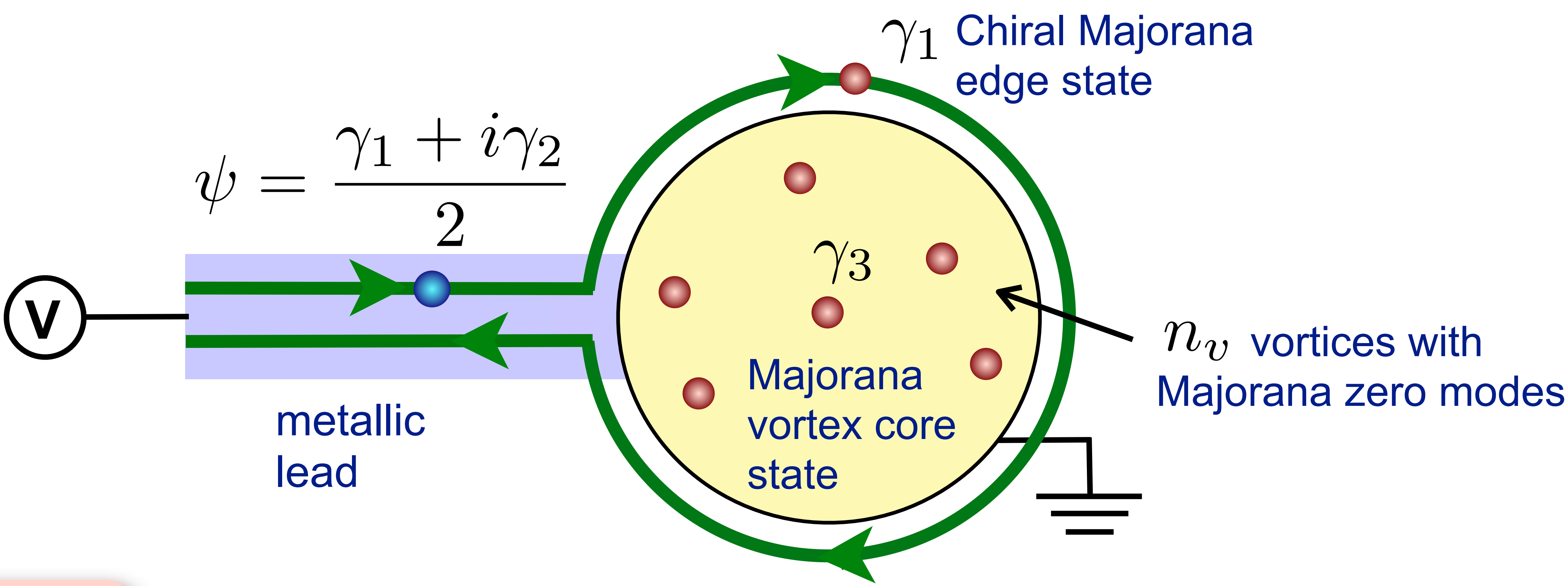
$$(U_{31})^2 = e^{-\frac{\pi}{2}\gamma_3\gamma_1}$$

State vector of injected electron (hole) : $|n_{12}\rangle$ $\qquad n_{12} = \psi^\dagger \psi$

γ_1 **travels around** γ_3 **and returns to original position**

$$(U_{31})^2|1\rangle = |0\rangle, \quad (U_{31})^2|0\rangle = -|1\rangle$$

Injected electron (hole) is perfectly converted to hole (electron) !!



n_v **: odd** ***injected electron (hole) is perfectly converted to hole (electron) !!***
(perfect Andreev reflection)

$$|1\rangle \rightarrow |0\rangle \qquad |0\rangle \rightarrow |1\rangle$$

Current : $I = \frac{2e}{h}\int_0^{eV} dE|s^{he}|^2$ **Conductance :** $G = 2\frac{e^2}{h}$ **irrespective of coupling between lead and SC**

n_v **: even** **Conductance :** $G = 0$ **(no Majorana edge state)**

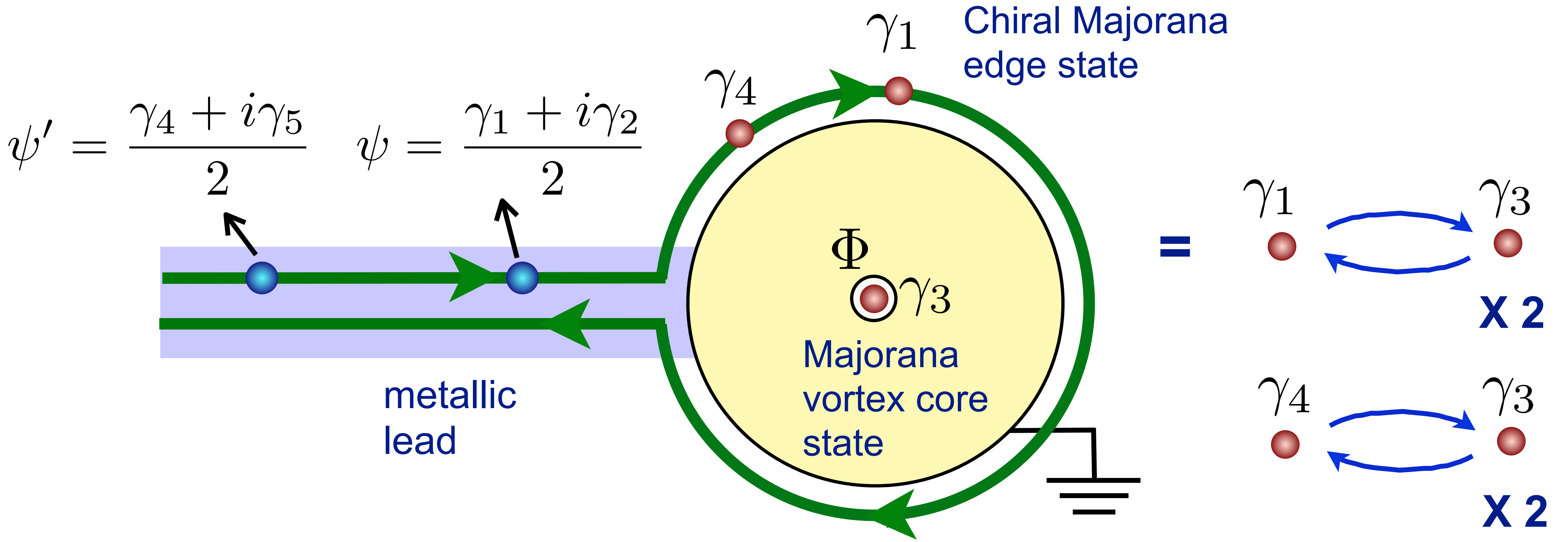
However, for realistic systems with multiple channels in the lead, electrons not coupled to chiral Majorana mode lead to non-quantized conductance

How to detect Non-Abelian statistics ?

Interferometer experiment II

vanishing of AB effect and AC effect

(Grosfeld, Stern; Stern,Halperin; Bonderson, Kitaev, Shtengel)

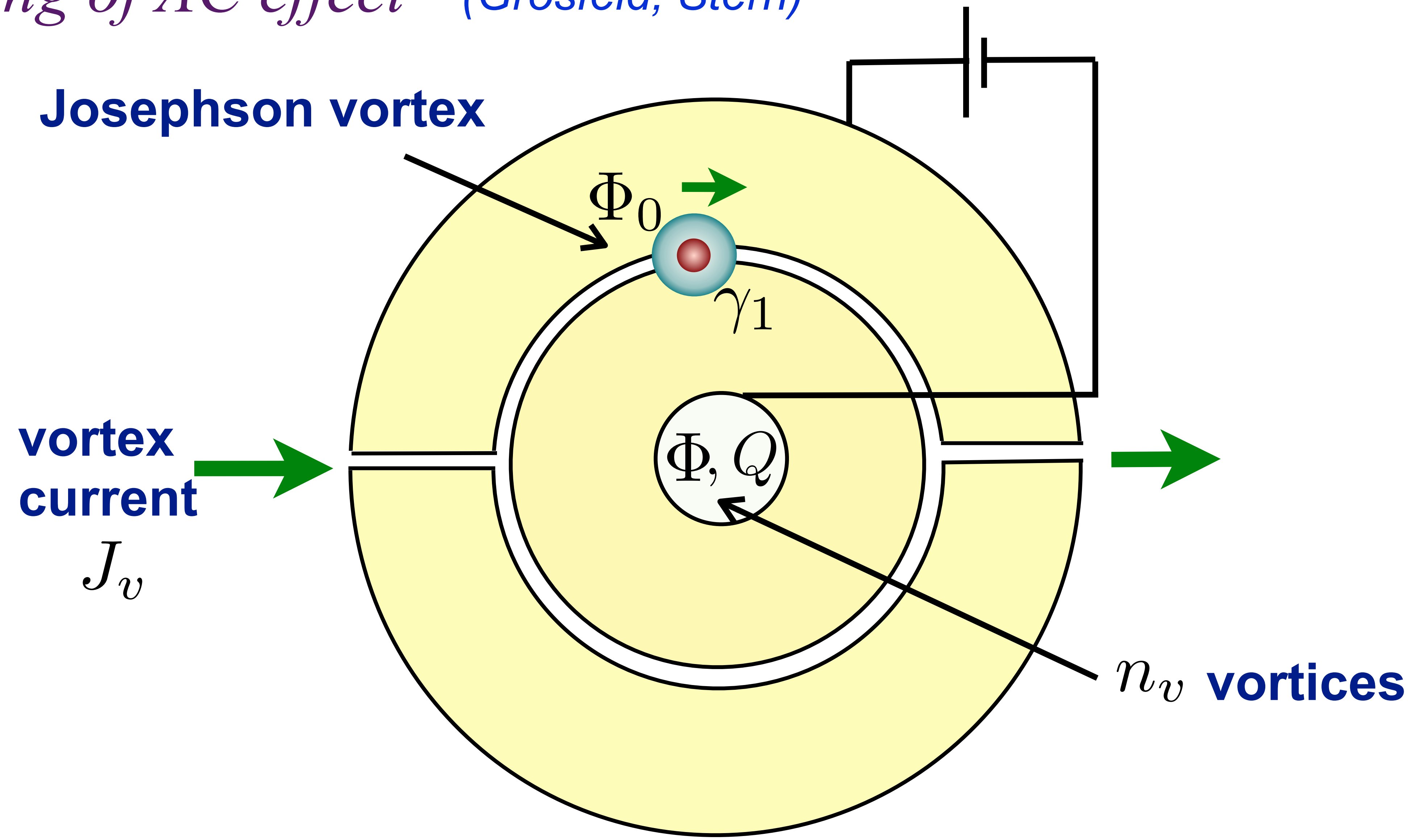


$(U_{31})^2 = \gamma_1\gamma_3$ **and** $(U_{43})^2 = \gamma_3\gamma_4$ **do not commute**

→ **dephasing of intereference**

→ **vanishing of AB effect**
(but experimental detection is not clear)

vanishing of AC effect (Grosfeld, Stern)



n_v : even

There is no M.F. in the center hole

conventional AC effect :

$$J_v \sim J_{v0} + J_{v1} \cos\left(2\pi \frac{Q}{2e}\right)$$

n_v : odd

M.F. in the center hole

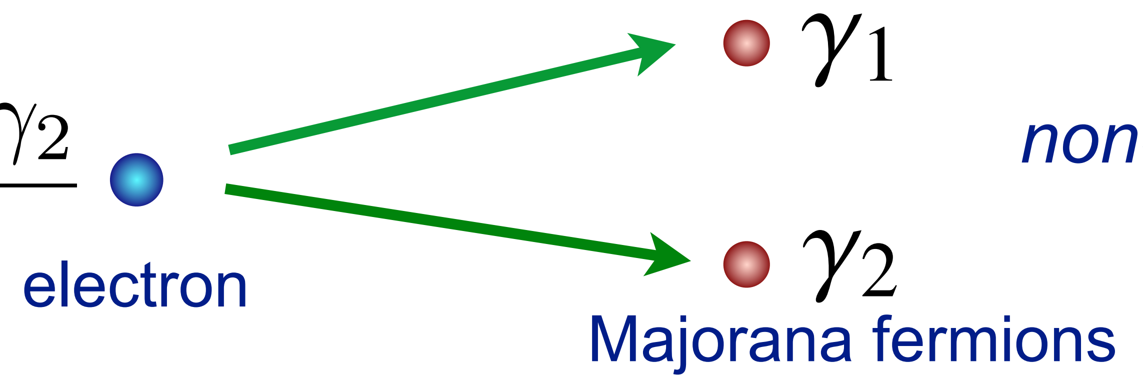
AC effect disappears !!

$$J_v \sim J_{v0}$$

Non-local correlation and “teleportation”

Splitting electrons into two M.F. and non-local correlation

$$\psi = \frac{\gamma_1 + i\gamma_2}{2}$$



non-local correlation ?

(Bolech,Demler; Tewari et al.; Semenoff; Nilsson,Akhmerov, Beenakker)

1D top. SC

Mode expansion of electron field :

$$\psi_\sigma(x) = \sum_{i=1,2} u_{\sigma i}(x)\gamma_i + (\text{non-zero energy modes})$$

correlation function of electrons :

$$\langle \psi_\sigma(x)\psi_\sigma^\dagger(y)\rangle \sim u_{\sigma 1}(x)u_{\sigma 2}^*(y)\underline{\langle\gamma_1\gamma_2\rangle}$$

$\langle\gamma_1\gamma_2\rangle \rightarrow \pm 1$

$x \sim 0$

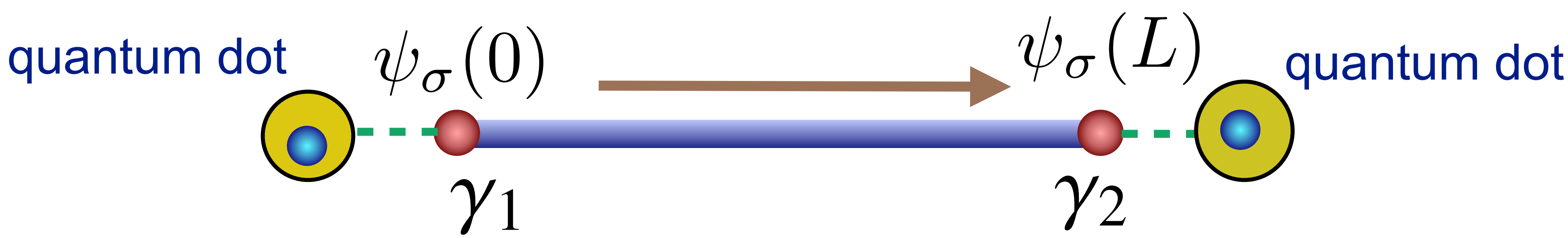
$y \sim L$

non-zero even for $|x - y| \rightarrow \infty$ ***non-local correlation !!***

non-local correlation independent of distance !!

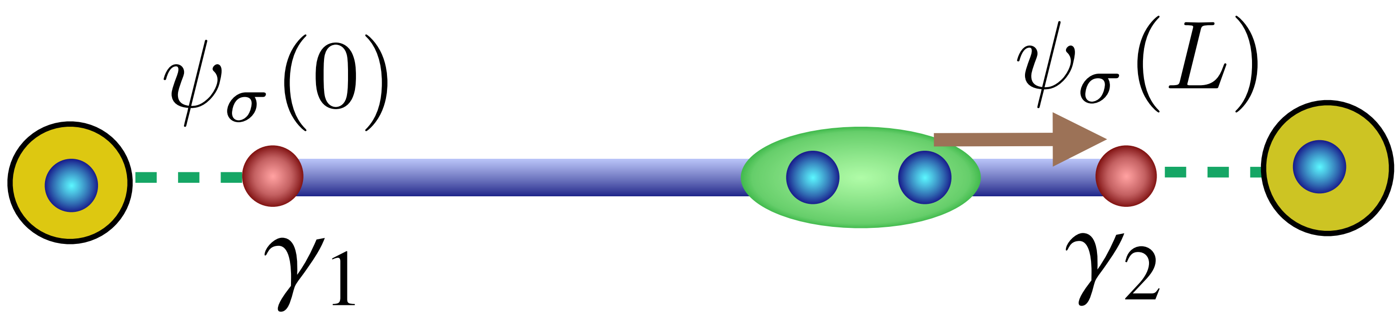
$$\langle \psi_\sigma(x)\psi^\dagger_\sigma(y)\rangle \sim u_{\sigma 1}(x)u^*_{\sigma 2}(y)\langle\gamma_1\gamma_2\rangle \neq 0 \quad \text{for } |x-y| \to \infty$$

→ **"teleportation" ?**



However ! problems arise !

(i) An electron detected at x=L may come from breaking up a Cooper pair



no energy cost, because zero-energy Majorana end states exist

(ii) Coupling with leads or dots to probe "teleportation" breaks Fermion-parity conservation

→ $\langle\gamma_1\gamma_2\rangle = 0$

vanishing correlation !!

(Bolech,Demler)

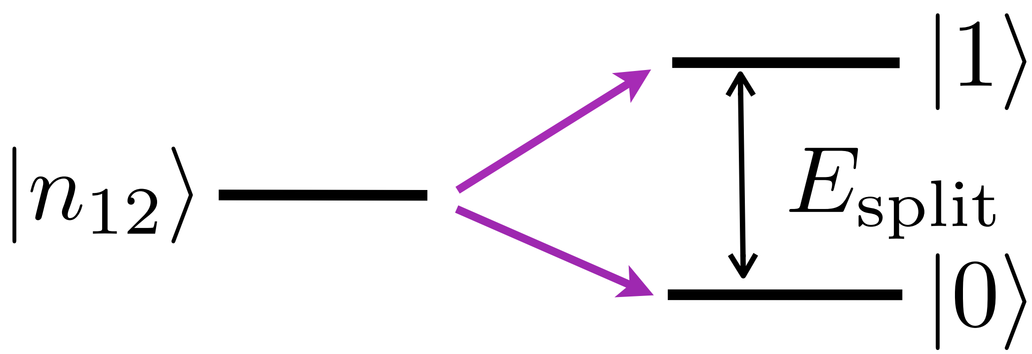
(ii) Coupling with leads or dots to probe "teleportation" breaks Fermion-parity conservation

$\Rightarrow \langle \gamma_1 \gamma_2 \rangle = 0$

vanishing correlation !!

(Bolech,Demler)

However, situation changes, when Fermion-parity degeneracy is lifted by overlap of Majorana-zero-mode wave functions.



$\langle \gamma_1 \gamma_2 \rangle = +1$ or -1

If $E_{\rm split}$ ***is larger than energy-scale of voltage applied on leads and T***

$$\langle \psi_\sigma(x) \psi_\sigma^\dagger(y) \rangle \sim u_{\sigma 1}(x) u_{\sigma 2}^*(y) \langle \gamma_1 \gamma_2 \rangle \neq 0$$

non-local correlation survives !!

(Nilsson,Akhmerov, Beenakker)

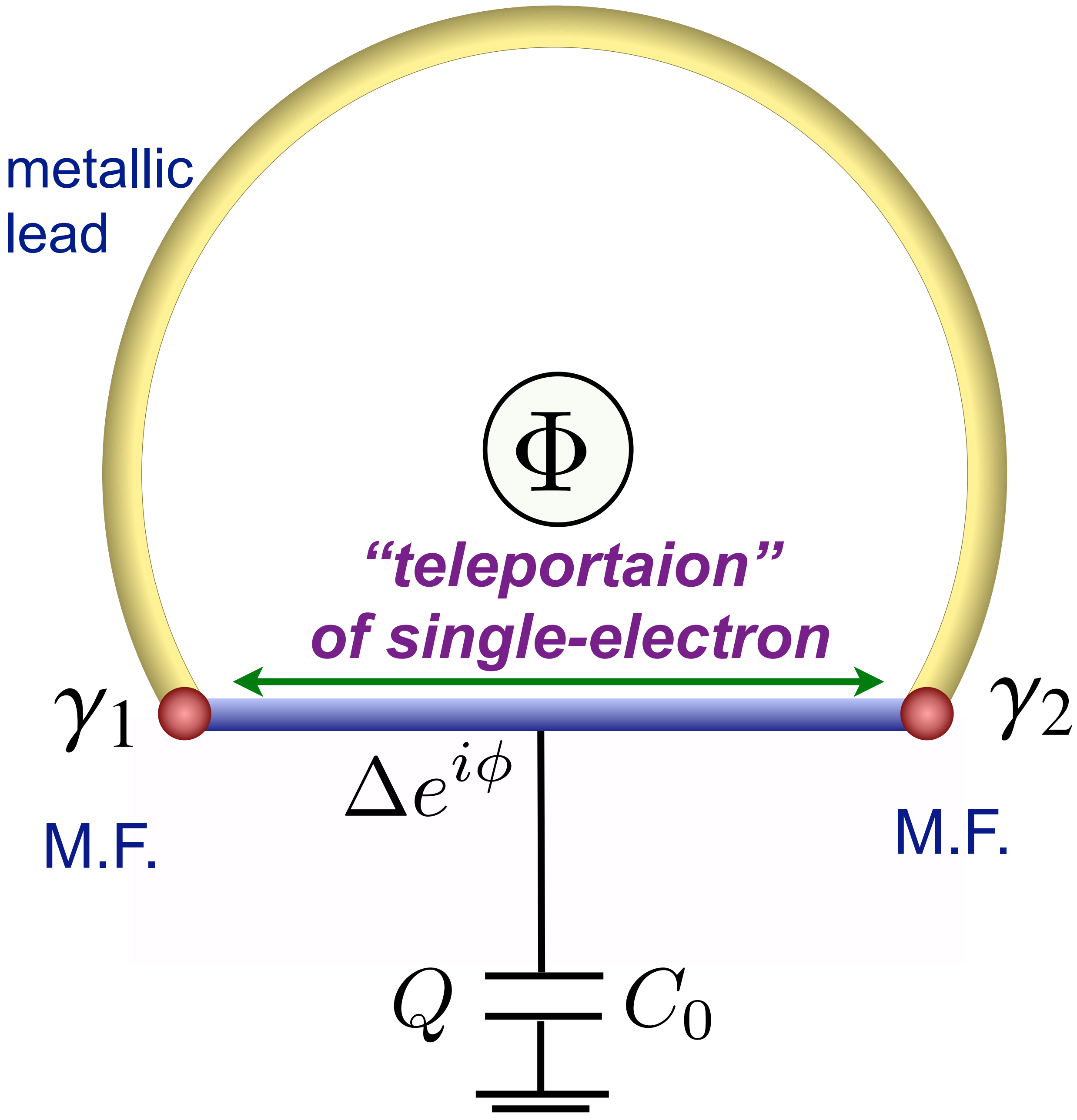
Correlation does not depend on |x-y| explicitly, though overlap does

Non-local correlation and "teleportation" in mesoscopic SC

1D top. SC

taking account of charging energy $Q^2/(2C_0)$
and fluctuation of SC phase ϕ

(Liang Fu)

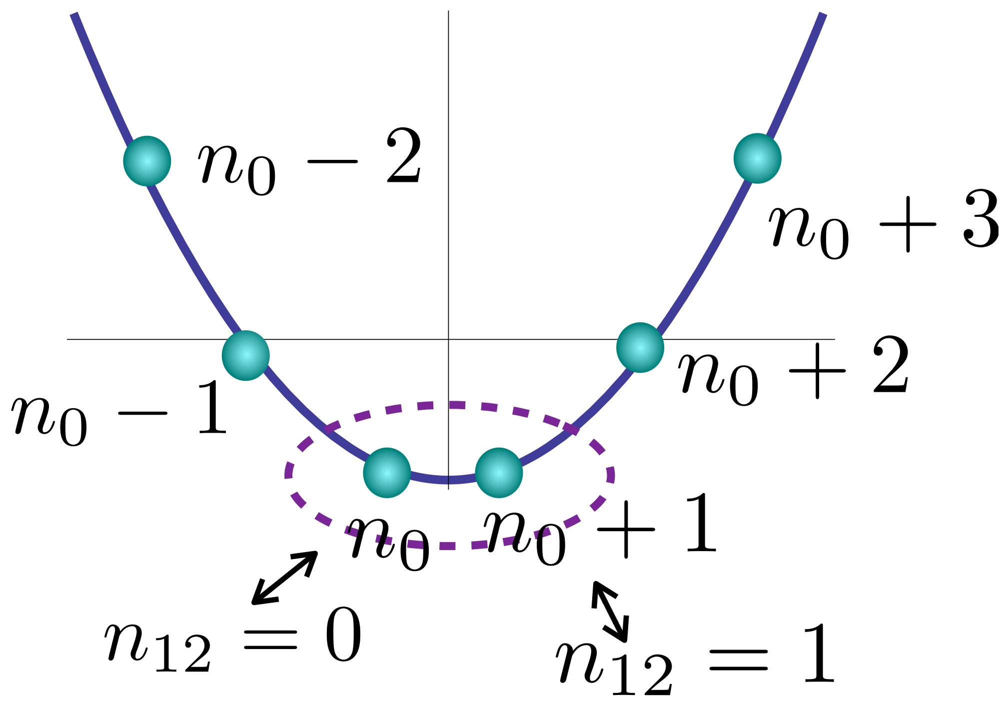


AB effect with period $\frac{h}{e}$

(not $\frac{h}{2e}$!!)

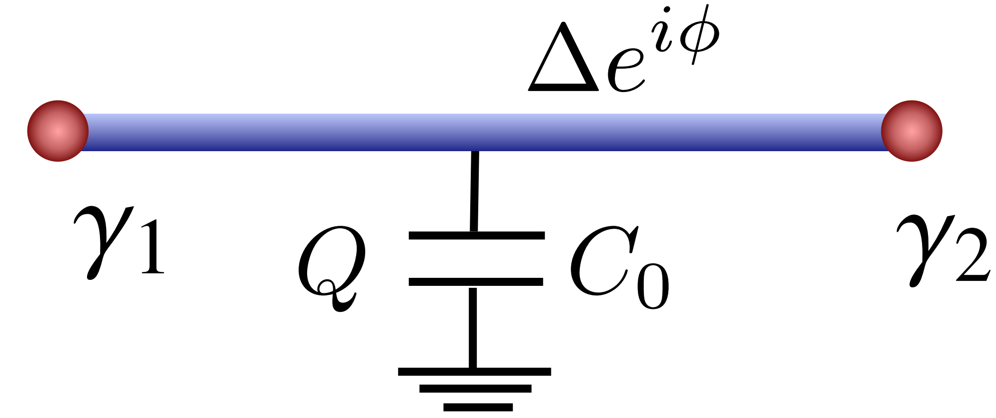
even for sufficiently large length of SC wire

charging energy : $Q^2/(2C_0)$



truncating Hilbert space

n_0 : # of electrons in SC



Fermion-parity degeneracy : $n_{12} = 0, \text{ or } 1$

$$n_{12} = \psi^\dagger_{12}\psi_{12} \qquad \psi_{12} = (\gamma_1 + i\gamma_2)/2$$

$$[n_{12}, e^{\pm i\frac{\phi}{2}}] = \pm e^{\pm i\frac{\phi}{2}}$$

(c.f. $[S^z, S^\pm] = \pm S^\pm$)

$e^{\pm i\frac{\phi}{2}}$ raising and lowering n_{12}

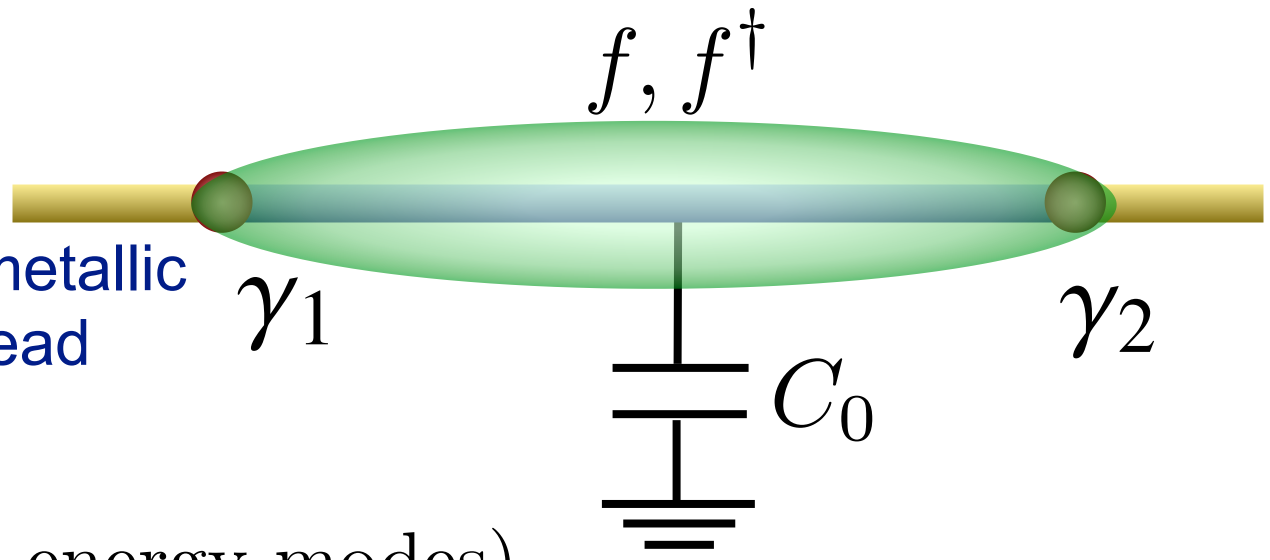
$$e^{i\frac{\phi}{2}}|0\rangle = |1\rangle, \quad e^{-i\frac{\phi}{2}}|1\rangle = |0\rangle,$$

$$e^{i\frac{\phi}{2}}|1\rangle = 0, \quad e^{-i\frac{\phi}{2}}|0\rangle = 0.$$

Furthermore, $e^{\pm i\frac{\phi}{2}}$ does not commute with Majorana fields γ_1, γ_2

$$[\gamma_1, e^{\pm i\frac{\phi}{2}}] = \pm(-1)^{n_{12}} \qquad [\gamma_2, e^{\pm i\frac{\phi}{2}}] = -i(-1)^{n_{12}}$$

Tunneling of electrons from leads into SC via M.F.



Mode expansion of electron field :

$$\psi_\sigma(x) = \sum_{i=1,2} u_{\sigma i}(x)\gamma_i e^{-i\frac{\phi}{2}} + (\text{non-zero energy modes})$$

Tunneling Hamiltonian at x=0 and *L* :

$$H_T = \sum_{k,\sigma}[V_{k\sigma 1}c^\dagger_{k\sigma}\gamma_1 e^{-i\frac{\phi}{2}} + V_{k\sigma 2}c^\dagger_{k\sigma}\gamma_2 e^{-i\frac{\phi}{2}} + V^*_{k\sigma 1}\gamma_1 c_{k\sigma}e^{i\frac{\phi}{2}} + V^*_{k\sigma 2}\gamma_2 c_{k\sigma}e^{i\frac{\phi}{2}}]$$

$$\Rightarrow \sum_{k,\sigma}\sqrt{2}[V_{k\sigma 1}c^\dagger_{k\sigma}f - i(-1)^{n_{12}}V_{k\sigma 2}c^\dagger_{k\sigma}f + V^*_{k\sigma 1}f^\dagger c_{k\sigma} + iV^*_{k\sigma 2}f^\dagger c_{k\sigma}(-1)^{n_{12}}].$$

independent of distance !!

We introduce an operator :

$$f = \frac{1}{\sqrt{2}}\gamma_1 e^{-i\frac{\phi}{2}}, \quad f^\dagger = \frac{1}{\sqrt{2}}e^{i\frac{\phi}{2}}\gamma_1$$

also, related to γ_2

$$\frac{1}{\sqrt{2}}\gamma_2 e^{-i\frac{\phi}{2}} = -i(-1)^{n_{12}}f, \quad \frac{1}{\sqrt{2}}e^{i\frac{\phi}{2}}\gamma_2 = if^\dagger(-1)^{n_{12}}$$

$$f = \frac{1}{\sqrt{2}}\gamma_1 e^{-i\frac{\phi}{2}}, \quad f^\dagger = \frac{1}{\sqrt{2}} e^{i\frac{\phi}{2}}\gamma_1$$

really conventional (complex) fermion or not ?

$$ff^\dagger + f^\dagger f = 1 \quad ✓$$

However $f^2 = 0, \quad (f^\dagger)^2 = 0$ ***hold only under a certain condition***

we need finite overlap between two M.F.

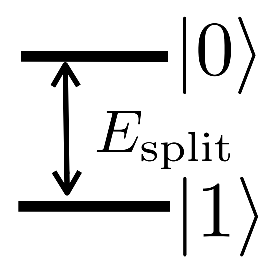
$$e^{i\frac{\phi}{2}}|0\rangle = |1\rangle, \quad e^{-i\frac{\phi}{2}}|1\rangle = |0\rangle,$$
$$e^{i\frac{\phi}{2}}|1\rangle = 0, \quad e^{-i\frac{\phi}{2}}|0\rangle = 0.$$
$$[\gamma_1, e^{\pm i\frac{\phi}{2}}] = \pm(-1)^{n_{12}}$$
$$[\gamma_2, e^{\pm i\frac{\phi}{2}}] = -i(-1)^{n_{12}}$$

$$f^2|0\rangle = 0, \quad (f^\dagger)^2|0\rangle = 0 \quad ✓$$

$$f^2|1\rangle = -\tfrac{1}{2}|1\rangle \quad (f^\dagger)^2|1\rangle = -\tfrac{1}{2}|1\rangle$$

When degeneracy is lifted by overlap, f ***is fermion within the space of*** $|0\rangle$

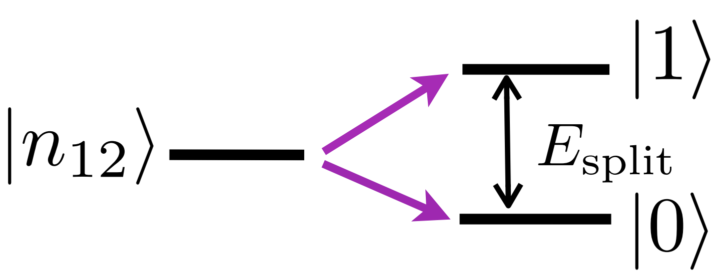
If



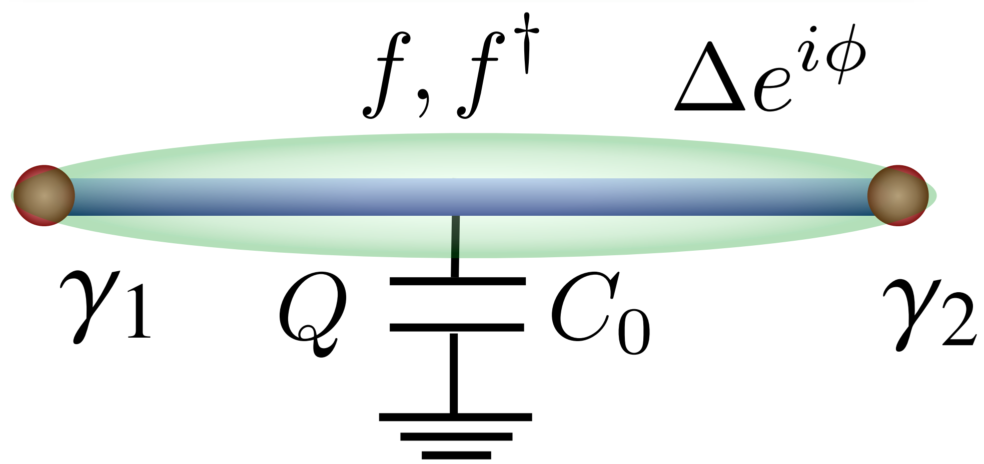
, definition of is changed to

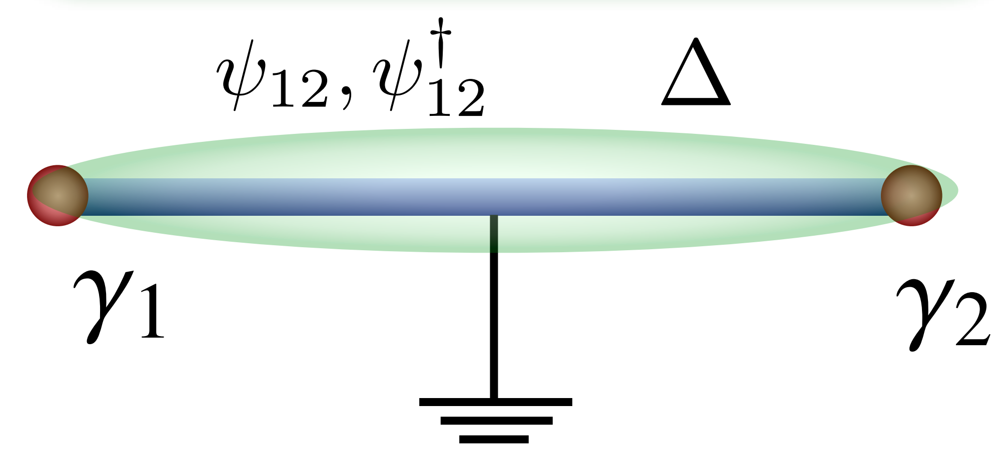
$$f = \frac{1}{\sqrt{2}} e^{-i\frac{\phi}{2}}\gamma_1$$

$$f^2|1\rangle = 0, \quad (f^\dagger)^2|1\rangle = 0$$



not grounded (with charging energy, phase fluctuation)	grounded (no charging energy, no phase fluctuation)





Tunneling Hamiltonian :

$$H_T = \sum_{k,\sigma}\sum_{i=1,2}[V_{k\sigma i}c^\dagger_{k\sigma}\gamma_i e^{-i\phi/2} + h.c.]$$

$$= \sum_{k,\sigma}\sqrt{2}[V_{k\sigma 1}c^\dagger_{k\sigma}f - i(-1)^{n_{12}}V_{k\sigma 2}c^\dagger_{k\sigma}f$$

$$+V^*_{k\sigma 1}f^\dagger c_{k\sigma} + iV^*_{k\sigma 2}f^\dagger c_{k\sigma}(-1)^{n_{12}}].$$

Tunneling Hamiltonian :

$$H_T = \sum_{k,\sigma}\sum_{i=1,2}[V_{k\sigma i}c^\dagger_{k\sigma}\gamma_i + h.c.]$$

$$= \sum_{k,\sigma}[V_{k\sigma}c^\dagger_{k\sigma}\psi_{12} + V_{k\sigma}c^\dagger_{k\sigma}\psi^\dagger_{12}$$

$$+V^*_{k\sigma}\psi^\dagger_{12}c_{k\sigma} + V^*_{k\sigma}\psi_{12}c_{k\sigma}]$$

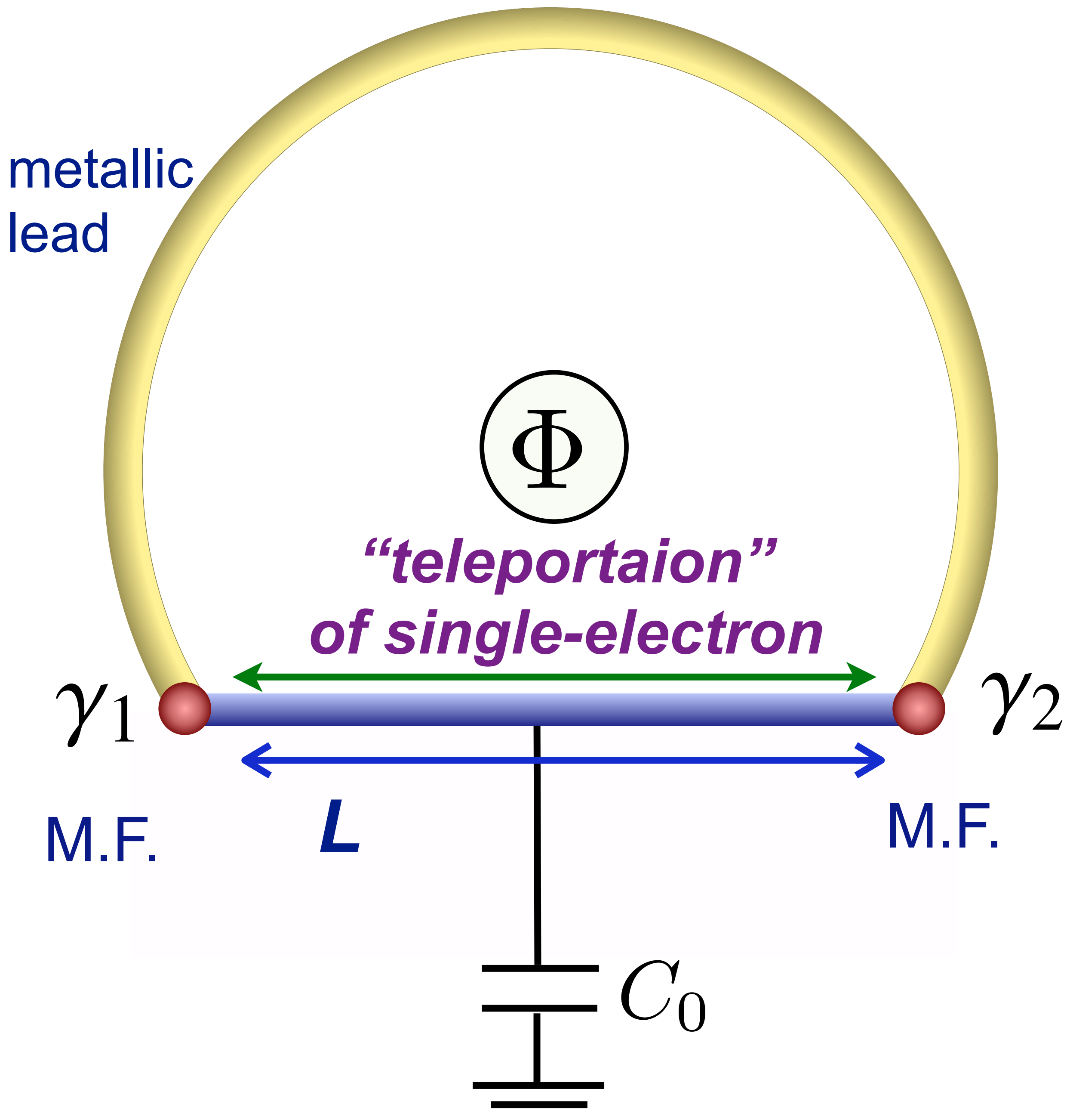
$$\psi_{12} = (\gamma_1 + i\gamma_2)/2$$

$$V_{k\sigma} = V_{k\sigma 1} - iV_{k\sigma 2}$$

Andreev scattering

AB effect due to "teleportation" via Majorana fermions

$$H_T = \sum_{k,\sigma} \sqrt{2}[V_{k\sigma 1} c^{\dagger}_{k\sigma} f - i(-1)^{n_{12}} V_{k\sigma 2} c^{\dagger}_{k\sigma} f + h.c.]$$



AB effect with period $\frac{h}{e}$

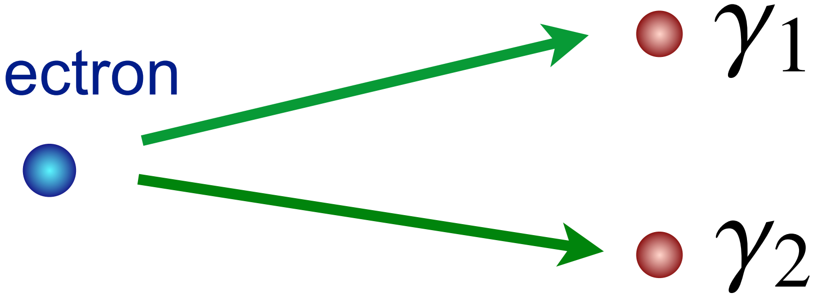
(not $\frac{h}{2e}$!!)

even for sufficiently large length of SC wire

Although level splitting is required, and it depends on the length L exponentially, non-local correlation does not depend on L explicitly !!

“Fractionalization” and 4π-periodic Josephson effect

$$\psi = \frac{\gamma_1 + i\gamma_2}{2}$$

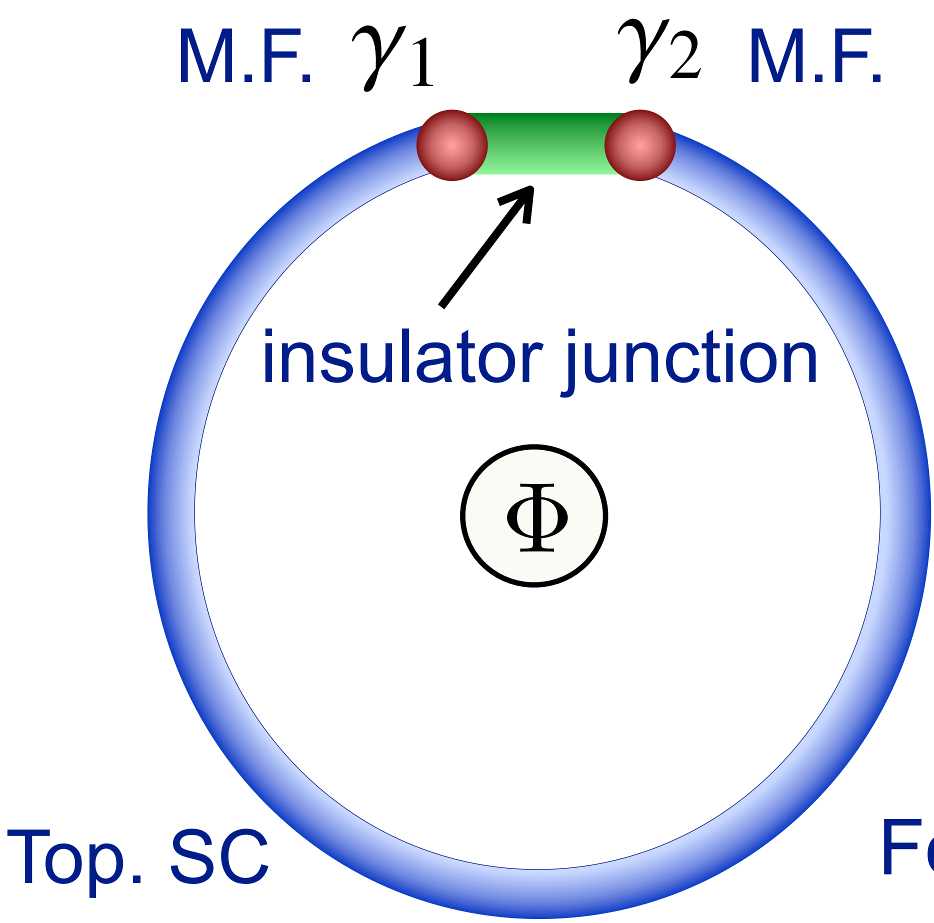


"Fractionalization"

Majorana fermions

4π-periodic (fractional) Josephson effect : Cooper pairs with $2e$ split into Cooper pairs with e

(Kwon, Senguputa, Yakovenko; Kitaev)



Single-electron tunneling Hamiltonian :

$$H_{1t} = -te^{i\phi}\psi^{\dagger}_{\sigma 1}\psi_{\sigma 2} + h.c. \qquad \phi = \frac{2e}{\hbar}\Phi$$

Mode expansion of electron fields: $i = 1, 2$

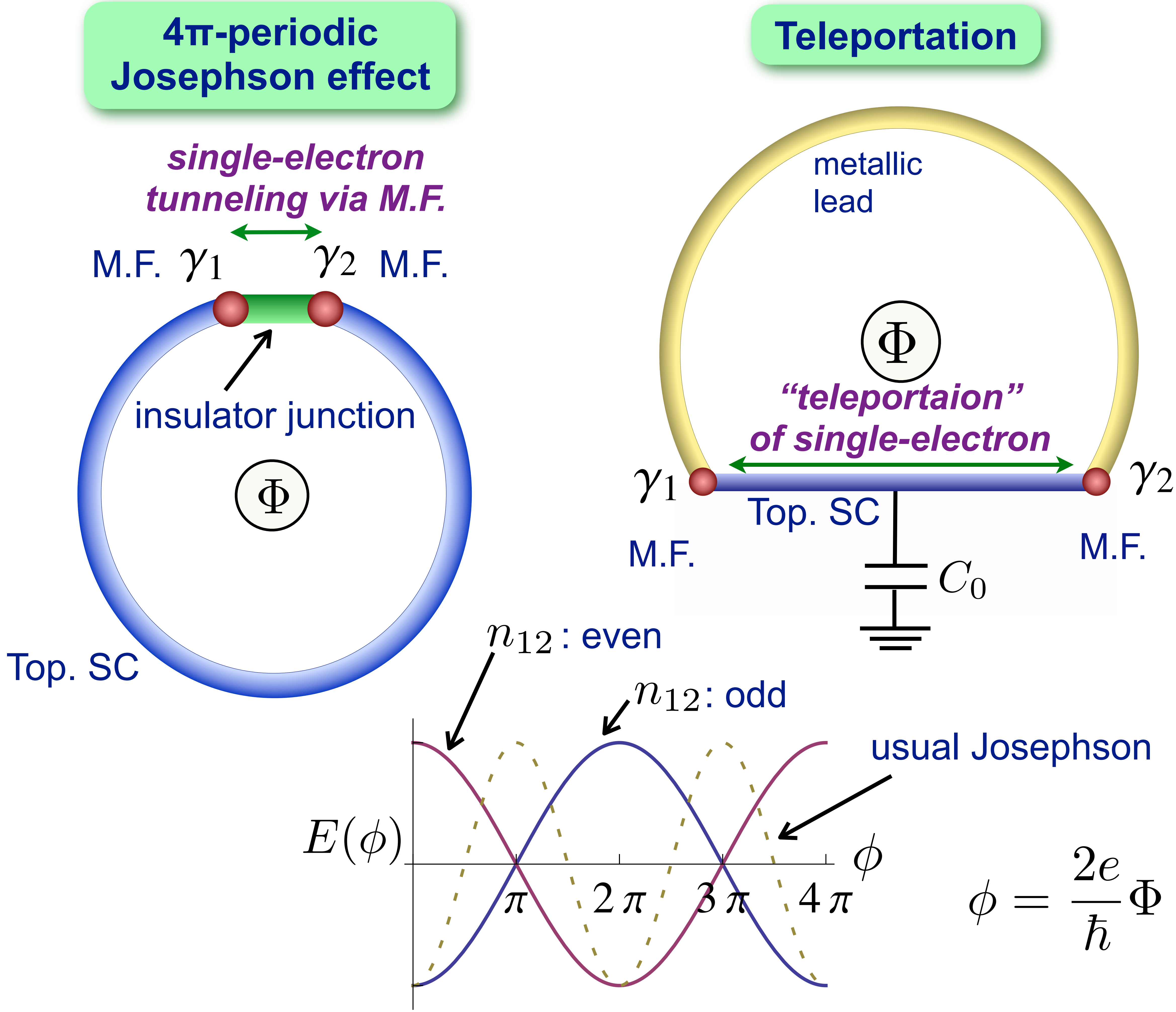
$$\psi_{\sigma i} = u_{\sigma i}(x)\gamma_i + \text{(non-zero energy modes)}$$

$$\Rightarrow \quad H_{1t} = J_M i\gamma_1\gamma_2 \cos\frac{\phi}{2}$$

For parity-eigen state $i\gamma_1\gamma_2 = 1, \text{ or } -1$

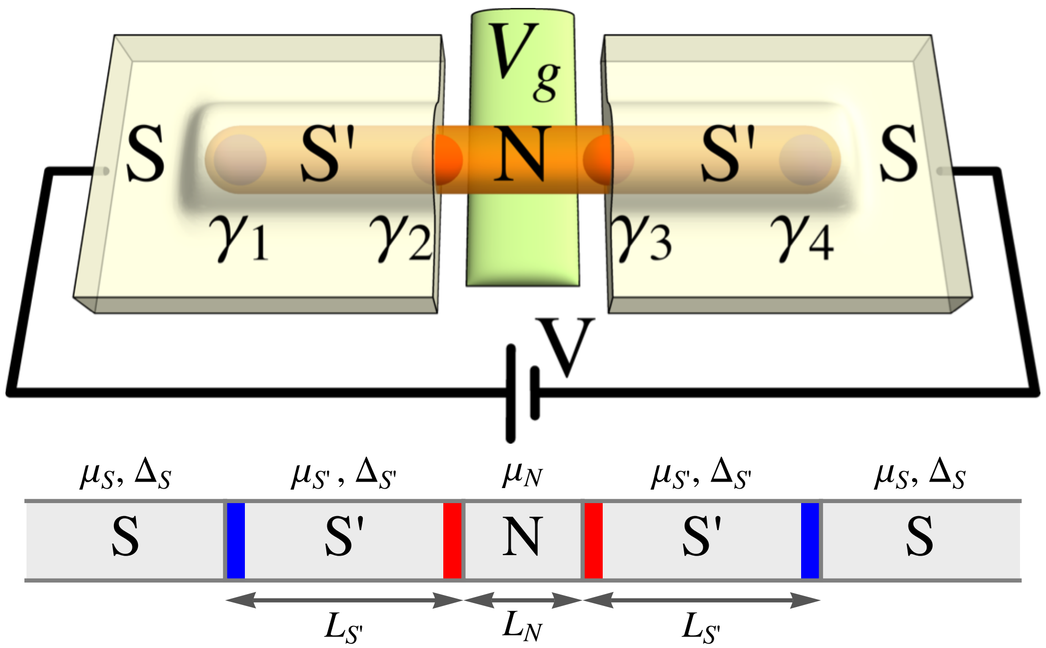
$\Rightarrow$ ***4π-periodic***

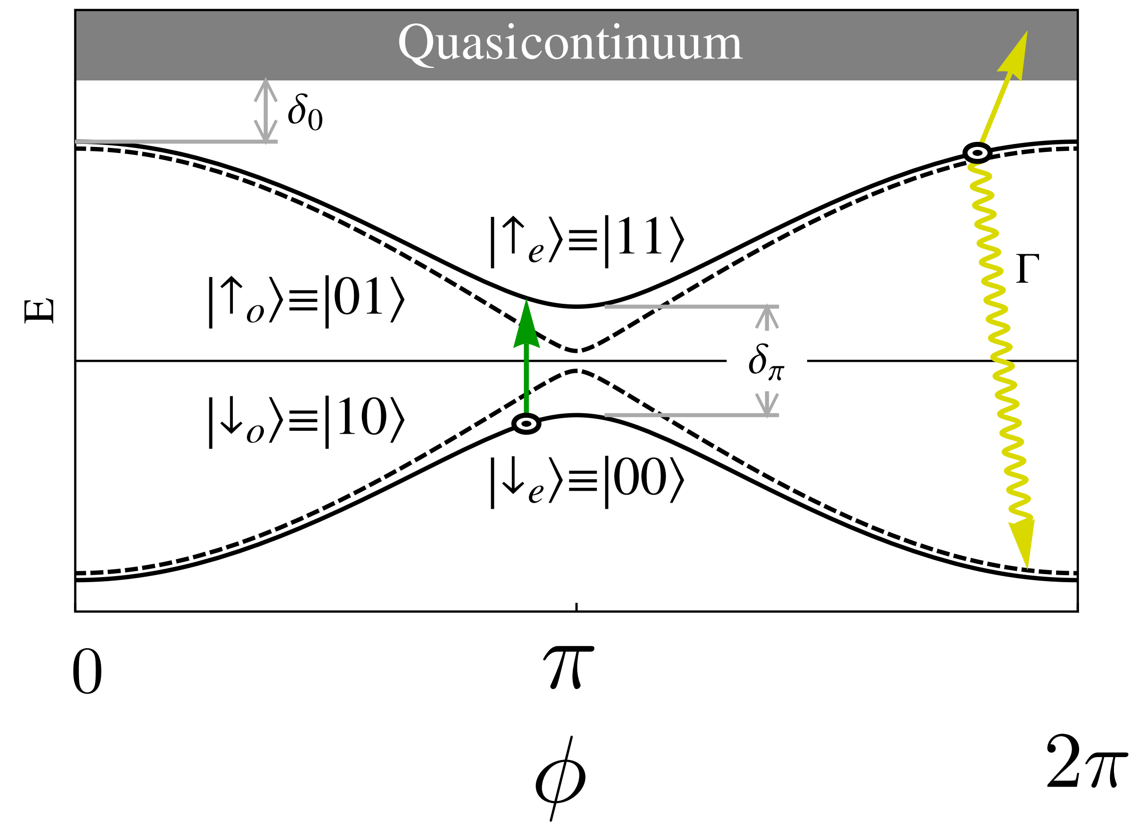
N.B. there is also usual Josephson tunneling with 2π-periodicity


4π-periodic Josephson effect
Teleportation
single-electron tunneling via M.F.
M.F. γ_1
γ_2 M.F.
insulator junction
Φ
Top. SC
metallic lead
Φ
"teleportaion" of single-electron
γ_1
γ_2
Top. SC
M.F.
M.F.
C_0
n_{12} : even
n_{12}: odd
usual Josephson
$E(\phi)$
ϕ
π
2π
3π
4π
$\phi = \frac{2e}{\hbar}\Phi$

AC 4π-periodic Josephson effect

(San-Jose et al.)





$$\psi_a = \gamma_2 + i\gamma_3 \qquad \psi_b = \gamma_1 + i\gamma_4$$

4π -periodic Josephson effect is absent for finite size systems, because of admixture with two other Majorana end states

However, ac 4π-periodic Josephson effect is still possible, because of non-adiabatic transition induced by ac fields

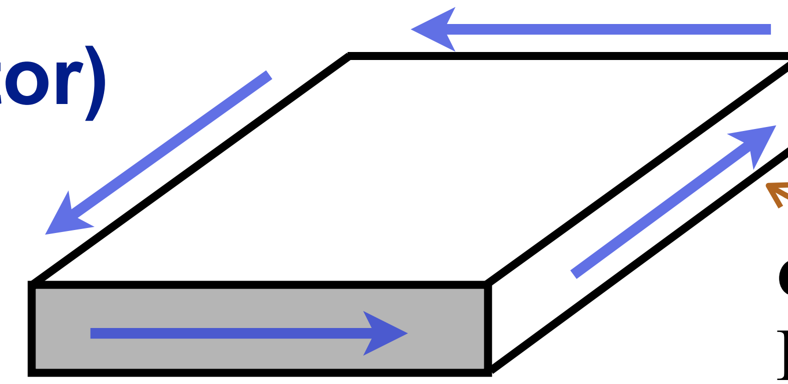
Experimentally detected ? Rokhinson et al., Nature Physics 8, 795 (2012)

not yet convincing

Thermal Responses

Chiral superconductor with broken TRS (class D and C)

Analogy to QHE (or Chern insulator)



QHE: $$\sigma_{xy} = \frac{e^2}{h} N$$

2D class D top. SC: *charge is not conserved*
but, energy is still conserved

$$\kappa_{xy} = \frac{\pi^2 k_B^2 T}{6h} N$$

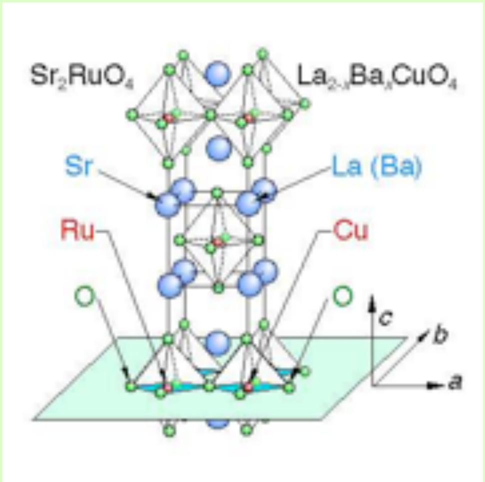
quantum anomalous thermal Hall effect

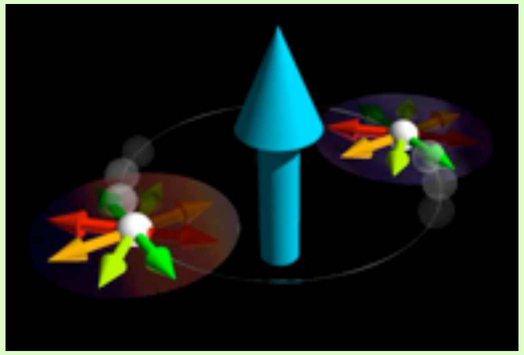
(Read, Green; Nomura et al.; Sumiyoshi, S.F.)

Sr_2RuO_4

Chiral p_x+ip_y SC (topological)

2D class D

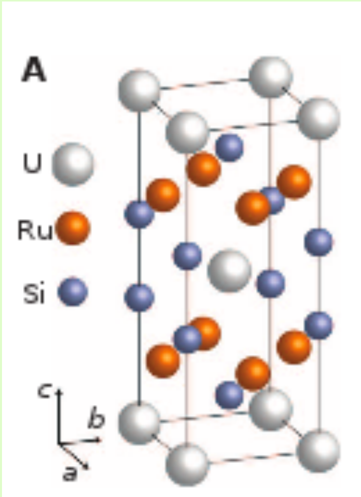




(Y. Maeno et al.)

URu_2Si_2

Chiral $d+id$ SC (not topological)



3D class C(trivial)

not quantized but still nonzero spontaneous thermal Hall effect

(Y. Kasahara et al.)

TRI topological superconductor (class DIII)

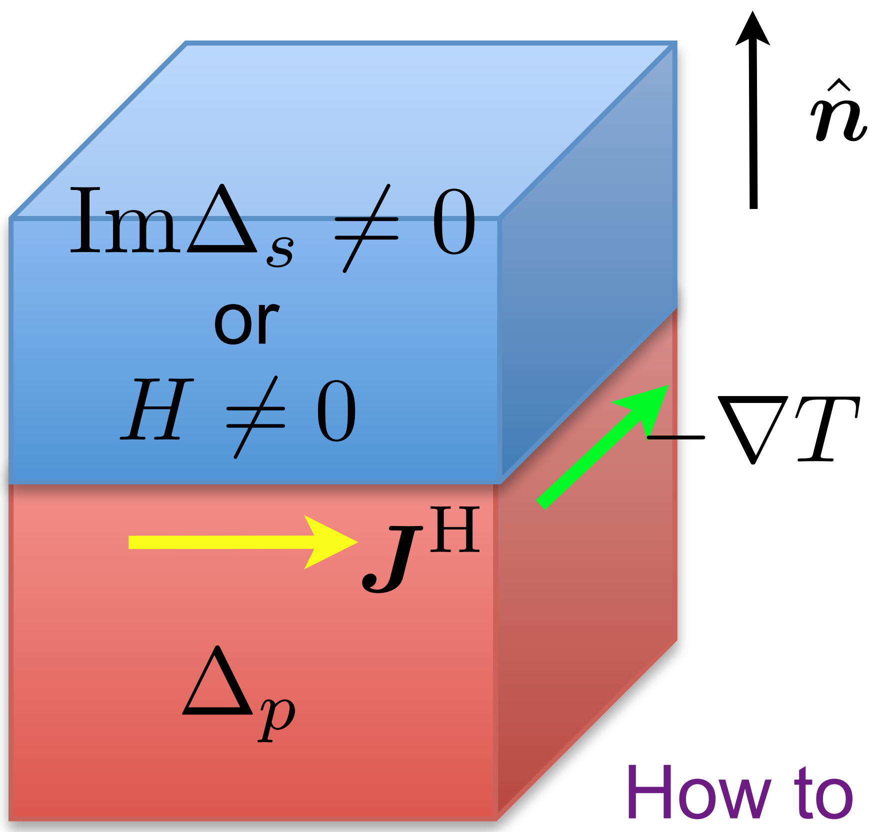
Quantum anomalous thermal Hall effect

$$J_H = -\frac{\pi^2 k_B^2 T}{12h} \underline{N} \hat{\boldsymbol{n}} \times \nabla T$$

(Wang, Qi, Zhang; Nomura et al.; Shiozaki, S.F.)

trivial TRB s-wave superconductor or ferromagnetic insulator

TRI topological superconductor



N is precisely equal to bulk top. invariant (winding number), when $\mathrm{Im}\Delta_s \neq 0$

How to realize phase difference ?

- bias between s-wave SC and TSC
- dynamical effect, $\mathrm{Im}\Delta_s(\omega) \neq 0$ for $\omega \neq 0$ due to inelastic scattering

SUMMARY

Exotic phenomena associated with Majorana fermions in SC

(i) Non-Abelian statistics

(ii) Non-local correlation and “teleportation”

(iii) Majorana fermion as “fractionalization” of electron

(iv) Thermal responses

In particular,
experimental detections of (i) and (ii) are the most important future issues